Chapter 7 Trigonometric Identities and Equations

7-1 Basic Trigonometric Identities

Page 427 Check for Understanding

1. Sample answer: $x = 45°$

2. Pythagorean identities are derived by applying the Pythagorean Theorem to a right triangle. The opposite angle identities are so named because $-A$ is the opposite of A.

3. $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\frac{\cos\theta}{\sin\theta} = \cot\theta$, $1 + \cot^2\theta = \csc^2\theta$

4. $\tan(-A) = \frac{\sin(-A)}{\cos(-A)}$
$= \frac{-\sin A}{\cos A}$
$= -\frac{\sin A}{\cos A}$
$= -\tan A$

5. Rosalinda is correct; there may be other values for which the equation is not true.

6. Sample answer: $\theta = 0°$
$\sin\theta + \cos\theta \stackrel{?}{=} \tan\theta$
$\sin 0° + \cos 0° \stackrel{?}{=} \tan 0°$
$0 + 1 \stackrel{?}{=} 0$
$1 \neq 0$

7. Sample answer: $x = 45°$
$\sec^2 x + \csc^2 x \stackrel{?}{=} 1$
$\sec^2 45° + \csc^2 45° \stackrel{?}{=} 1$
$(\sqrt{2})^2 + (\sqrt{2})^2 \stackrel{?}{=} 1$
$2 + 2 \stackrel{?}{=} 1$
$4 \neq 1$

8. $\sec\theta = \frac{1}{\cos\theta}$
$\sec\theta = \frac{1}{\frac{2}{3}}$
$\sec\theta = \frac{3}{2}$

9. $\tan\theta = \frac{1}{\cot\theta}$
$\tan\theta = \frac{1}{-\frac{\sqrt{5}}{2}}$
$\tan\theta = -\frac{2}{\sqrt{5}}$
$\tan\theta = -\frac{2\sqrt{5}}{5}$

10. $\sin^2\theta + \cos^2\theta = 1$
$\left(-\frac{1}{5}\right)^2 + \cos^2\theta = 1$
$\frac{1}{25} + \cos^2\theta = 1$
$\cos^2\theta = \frac{24}{25}$
$\cos\theta = \pm\frac{2\sqrt{6}}{5}$
Quadrant III, so $-\frac{2\sqrt{6}}{5}$

11. $\tan^2\theta + 1 = \sec^2\theta$
$\left(-\frac{4}{7}\right)^2 + 1 = \sec^2\theta$
$\frac{16}{49} + 1 = \sec^2\theta$
$\frac{65}{49} = \sec^2\theta$
$\pm\frac{\sqrt{65}}{7} = \sec\theta$
Quadrant IV, so $\frac{\sqrt{65}}{7}$

12. $\frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$
$\cos\frac{7\pi}{3} = \cos\left(2\pi + \frac{\pi}{3}\right)$
$= \cos\frac{\pi}{3}$

13. $-330° = -360° + 30°$
$\csc(-330°) = \frac{1}{\sin(-330°)}$
$= \frac{1}{\sin(-360° + 30°)}$
$= \frac{1}{\sin 30°}$
$= \csc 30°$

14. $\frac{\csc\theta}{\cot\theta} = \frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$
$= \frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$
$= \frac{1}{\cos\theta}$
$= \sec\theta$

15. $\cos x \csc x \tan x = \cos x\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)$
$= 1$

16. $\cos x \cot x + \sin x = \cos x\left(\frac{\cos x}{\sin x}\right) + \sin x$
$= \frac{\cos^2 x}{\sin x} + \sin x$
$= \frac{\cos^2 x + \sin^2 x}{\sin x}$
$= \frac{1}{\sin x}$
$= \csc x$

17. $B = \frac{F\csc\theta}{I\ell}$
$BI\ell = F\csc\theta$
$F = \frac{BI\ell}{\csc\theta}$
$F = BI\ell\left(\frac{1}{\csc\theta}\right)$
$F = BI\ell\sin\theta$

Pages 427–430 Exercises

18. Sample answer: 45°
$\sin\theta\cos\theta \stackrel{?}{=} \cot\theta$
$\sin 45°\cos 45° \stackrel{?}{=} \cot 45°$
$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \stackrel{?}{=} 1$
$\frac{1}{2} \neq 1$

19. Sample answer: 45°
$\frac{\sec\theta}{\tan\theta} \stackrel{?}{=} \sin\theta$
$\frac{\sec 45°}{\tan 45°} \stackrel{?}{=} \sin 45°$
$\frac{\sqrt{2}}{1} \stackrel{?}{=} \frac{\sqrt{2}}{2}$
$\sqrt{2} \neq \frac{\sqrt{2}}{2}$

20. Sample answer: 30°

$$\sec^2 x - 1 \stackrel{?}{=} \frac{\cos x}{\csc x}$$
$$\sec^2 30° - 1 \stackrel{?}{=} \frac{\cos 30°}{\csc 30°}$$
$$\left(\frac{2\sqrt{3}}{3}\right)^2 - 1 \stackrel{?}{=} \frac{\frac{\sqrt{3}}{2}}{2}$$
$$\frac{12}{9} - 1 \stackrel{?}{=} \frac{\sqrt{3}}{4}$$
$$\frac{1}{3} \neq \frac{\sqrt{3}}{4}$$

21. Sample answer: 30°

$$\sin x + \cos x \stackrel{?}{=} 1$$
$$\sin 30° + \cos 30° \stackrel{?}{=} 1$$
$$\frac{1}{2} + \frac{\sqrt{3}}{2} \stackrel{?}{=} 1$$
$$\frac{1 + \sqrt{3}}{2} \neq 1$$

22. Sample answer: 0°

$$\sin y \tan y \stackrel{?}{=} \cos y$$
$$\sin 0° \tan 0° \stackrel{?}{=} \cos 0°$$
$$0 \cdot 0 \stackrel{?}{=} 1$$
$$0 \neq 1$$

23. Sample answer: 45°

$$\tan^2 A + \cot^2 A \stackrel{?}{=} 1$$
$$\tan^2 45° + \cot^2 45° \stackrel{?}{=} 1$$
$$1 + 1 \stackrel{?}{=} 1$$
$$2 \neq 1$$

24. Sample answer: 0

$$\cos\left(\theta + \frac{\pi}{2}\right) \neq \cos\theta + \cos\frac{\pi}{2}$$
$$\cos\left(0 + \frac{\pi}{2}\right) \neq \cos 0 + \cos\frac{\pi}{2}$$
$$\cos\frac{\pi}{2} \neq \cos 0 + \cos\frac{\pi}{2}$$
$$0 \neq 1 + 0$$
$$0 \neq 1$$

25. $\csc\theta = \frac{1}{\sin\theta}$

$$\csc\theta = \frac{1}{\frac{2}{5}}$$
$$\csc\theta = \frac{5}{2}$$

26. $\cot\theta = \frac{1}{\tan\theta}$

$$\cot\theta = \frac{1}{\frac{\sqrt{3}}{4}}$$
$$\cot\theta = \frac{4}{\sqrt{3}}$$
$$\cot\theta = \frac{4\sqrt{3}}{3}$$

27. $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1$$
$$\frac{1}{16} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{15}{16}$$
$$\cos\theta = \pm\frac{\sqrt{15}}{4}$$

Quadrant I, so $\frac{\sqrt{15}}{4}$

28. $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \left(-\frac{2}{3}\right)^2 = 1$$
$$\sin^2\theta + \frac{4}{9} = 1$$
$$\sin^2\theta = \frac{5}{9}$$
$$\sin\theta = \pm\frac{\sqrt{5}}{3}$$

Quadrant II, so $\frac{\sqrt{5}}{3}$

29. $1 + \cot^2\theta = \csc^2\theta$

$$1 + \cot^2\theta = \left(\frac{\sqrt{11}}{3}\right)^2$$
$$1 + \cot^2\theta = \frac{11}{9}$$
$$\cot^2\theta = \frac{2}{9}$$
$$\cot\theta = \pm\frac{\sqrt{2}}{3}$$

Quadrant II, so $-\frac{\sqrt{2}}{3}$

30. $\tan^2\theta + 1 = \sec^2\theta$

$$\tan^2\theta + 1 = \left(-\frac{5}{4}\right)^2$$
$$\tan^2\theta + 1 = \frac{25}{16}$$
$$\tan^2\theta = \frac{9}{16}$$
$$\tan\theta = \pm\frac{3}{4}$$

Quadrant II, so $-\frac{3}{4}$

31. $\sin^2\theta + \cos^2\theta = 1$

$$\left(-\frac{1}{3}\right)^2 + \cos^2\theta = 1$$
$$\frac{1}{9} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{8}{9}$$
$$\cos\theta = \pm\frac{2\sqrt{2}}{3}$$

Quadrant III, so $\cos\theta = -\frac{2\sqrt{2}}{3}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\tan\theta = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}}$$
$$\tan\theta = \frac{1}{2\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}}{4}$$

32. $\tan^2\theta + 1 = \sec^2\theta$

$$\left(\frac{2}{3}\right)^2 + 1 = \sec^2\theta$$
$$\frac{4}{9} + 1 = \sec^2\theta$$
$$\frac{13}{9} = \sec^2\theta$$
$$\pm\frac{\sqrt{13}}{3} = \sec\theta$$

Quadrant III, so $\sec\theta = -\frac{\sqrt{13}}{3}$

$$\cos\theta = \frac{1}{\sec\theta}$$
$$\cos\theta = \frac{1}{\frac{-\sqrt{13}}{3}}$$
$$\cos\theta = -\frac{3}{\sqrt{13}} \text{ or } -\frac{3\sqrt{13}}{13}$$

33. $\cos\theta = \frac{1}{\sec\theta}$

$$\cos\theta = \frac{1}{-\frac{7}{5}}$$
$$\cos\theta = -\frac{5}{7}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(-\frac{5}{7}\right)^2 = 1$$
$$\sin^2\theta + \frac{25}{49} = 1$$
$$\sin^2\theta = \frac{24}{49}$$
$$\sin\theta = \pm\frac{2\sqrt{6}}{7}$$

Quadrant III, so $-\frac{2\sqrt{6}}{7}$

34. $\sec\theta = \frac{1}{\cos\theta}$

$\sec\theta = \frac{1}{\frac{1}{8}}$

$\sec\theta = 8$

$\tan^2\theta + 1 = \sec^2\theta$
$\tan^2\theta + 1 = 8^2$
$\tan^2\theta + 1 = 64$
$\tan^2\theta = 63$
$\tan\theta = \pm 3\sqrt{7}$

Quadrant IV, so $-3\sqrt{7}$

35. $1 + \cot^2\theta = \csc^2\theta$

$1 + \left(-\frac{4}{3}\right)^2 = \csc^2\theta$

$1 + \frac{16}{9} = \csc^2\theta$

$\frac{25}{9} = \csc^2\theta$

$\pm\frac{5}{3} = \csc\theta$

$\sin\theta = \frac{1}{\csc\theta}$

$\sin\theta = \frac{1}{-\frac{5}{3}}$

$\sin\theta = -\frac{3}{5}$

Quadrant IV, so $-\frac{5}{3}$

36. $1 + \cot^2\theta = \csc^2\theta$
$1 + (-8)^2 = \csc^2\theta$
$1 + 64 = \csc^2\theta$
$65 = \csc^2\theta$
$\pm\sqrt{65} = \csc\theta$
Quadrant IV, so $-\sqrt{65}$

37. $\sec\theta = \frac{1}{\cos\theta}$

$\sec\theta = \frac{1}{-\frac{\sqrt{3}}{4}}$

$\sec\theta = -\frac{4}{\sqrt{3}}$ or $-\frac{4\sqrt{3}}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1$

$\sin^2\theta + \frac{3}{16} = 1$

$\sin^2\theta = \frac{13}{16}$

$\sin\theta = \pm\frac{\sqrt{13}}{4}$

Quadrant II, so $\frac{\sqrt{13}}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{\frac{\sqrt{13}}{4}}{-\frac{\sqrt{3}}{4}}$

$\tan\theta = -\frac{\sqrt{13}}{\sqrt{3}}$ or $-\frac{\sqrt{39}}{3}$

$$\frac{\sec^2 A - \tan^2 A}{2\sin^2 A + 2\cos^2 A} = \frac{\left(-\frac{4\sqrt{3}}{3}\right)^2 - \left(-\frac{\sqrt{39}}{3}\right)^2}{2\left(\frac{\sqrt{13}}{4}\right)^2 + 2\left(-\frac{\sqrt{3}}{4}\right)^2}$$

$$= \frac{\frac{48}{9} - \frac{39}{9}}{2\left(\frac{13}{16}\right) + 2\left(\frac{3}{16}\right)}$$

$$= \frac{\frac{9}{9}}{\frac{32}{16}}$$

$$= \frac{1}{2}$$

38. $390° = 360° + 30°$
$\sin 390° = \sin(360° + 30°)$
$= \sin 30°$

39. $\frac{27\pi}{8} = 3\pi + \frac{3\pi}{8}$

$\cos\frac{27\pi}{8} = \cos\left(3\pi + \frac{3\pi}{8}\right)$

$= -\cos\frac{3\pi}{8}$

40. $\frac{19\pi}{5} = 2(2\pi) - \frac{\pi}{5}$

$$\tan\frac{19\pi}{5} = \frac{\sin\frac{19\pi}{5}}{\cos\frac{19\pi}{5}}$$

$$= \frac{\sin\left(2(2\pi) - \frac{\pi}{5}\right)}{\cos\left(2(2\pi) - \frac{\pi}{5}\right)}$$

$$= \frac{-\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}}$$

$$= -\tan\frac{\pi}{5}$$

41. $\frac{10\pi}{3} = 3\pi + \frac{\pi}{3}$

$$\csc\frac{10\pi}{3} = \frac{1}{\sin\frac{10\pi}{3}}$$

$$= \frac{1}{\sin\left(3\pi + \frac{\pi}{3}\right)}$$

$$= \frac{1}{-\sin\frac{\pi}{3}}$$

$$= -\csc\frac{\pi}{3}$$

42. $-1290° = -7(180°) - 30°$

$$\sec(-1290°) = \frac{1}{\cos(-1290°)}$$

$$= \frac{1}{\cos(-7(180°) - 30°)}$$

$$= \frac{1}{-\cos 30°}$$

$$= -\sec 30°$$

43. $-660° = -2(360°) + 60°$

$$\cot(-660°) = \frac{\cos(-660°)}{\sin(-660°)}$$

$$= \frac{\cos(-2(360°) + 60°)}{\sin(-2(360°) + 60°)}$$

$$= \frac{\cos 60°}{\sin 60°}$$

$$= \cot 60°$$

44. $$\frac{\sec x}{\tan x} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

45. $$\frac{\cot\theta}{\cos\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta$$

46. $$\frac{\sin(\theta + \pi)}{\cos(\theta - \pi)} = \frac{-\sin\theta}{-\cos\theta}$$

$$= \tan\theta$$

47. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$
$= \sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x - 2\sin x\cos x + \cos^2 x$
$= 2\sin^2 x + 2\cos^2 x$
$= 2(\sin^2 x + \cos^2 x)$
$= 2$

48. $\sin x \cos x \sec x \cot x = \sin x \cos x\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)$
$= \cos x$

49. $\cos x \tan x + \sin x \cot x = \cos x\left(\frac{\sin x}{\cos x}\right) + \sin x\left(\frac{\cos x}{\sin x}\right)$
$= \sin x + \cos x$

50. $(1 + \cos\theta)(\csc\theta - \cot\theta) = (1 + \cos\theta)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)$
$= (1 + \cos\theta)\left(\frac{1 - \cos\theta}{\sin\theta}\right)$
$= \frac{1 - \cos^2\theta}{\sin\theta}$
$= \frac{\sin^2\theta}{\sin\theta}$
$= \sin\theta$

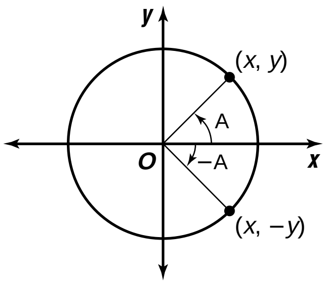
51. $1 + \cot^2\theta - \cos^2\theta - \cos^2\theta\cot^2\theta$
$= 1 + \cot^2\theta - \cos^2\theta(1 + \cot^2\theta)$
$= \csc^2\theta - \cos^2\theta(\csc^2\theta)$
$= \csc^2\theta(1 - \cos^2\theta)$
$= \csc^2\theta(\sin^2\theta)$
$= \frac{1}{\sin^2\theta}(\sin^2\theta)$
$= 1$

52. $\frac{\sin x}{1 + \cos x} + \frac{\sin x}{1 - \cos x}$
$= \frac{\sin x - \sin x\cos x}{1 - \cos^2 x} + \frac{\sin x + \sin x\cos x}{1 - \cos^2 x}$
$= \frac{2\sin x}{1 - \cos^2 x}$
$= \frac{2\sin x}{\sin^2 x}$
$= \frac{2}{\sin x}$
$= 2\csc x$

53. $\cos^4\alpha + 2\cos^2\alpha\sin^2\alpha + \sin^4\alpha = (\cos^2\alpha + \sin^2\alpha)^2$
$= 1^2$ or 1

54. $I = I_0\cos^2\theta$
$0 = I_0\cos^2\theta$
$0 = \cos^2\theta$
$0 = \cos\theta$
$\cos^{-1} 0 = \theta$
$90° = \theta$

55. Let (x, y) be the point where the terminal side of A intersects the unit circle when A is in standard position. When A is reflected about the x-axis to obtain $-A$, the y-coordinate is multiplied by -1, but the x-coordinate is unchanged. So,
$\sin(-A) = -y = -\sin A$ and
$\cos(-A) = x = \cos A$.

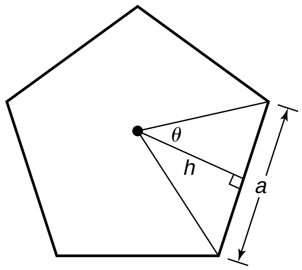


56a. $e = \frac{W\sec\theta}{As}$
$eAs = W\sec\theta$
$\frac{eAs}{\sec\theta} = W$
$W = eAs\cos\theta$

56b. $W = eAs\cos\theta$
$W = 0.80(0.75)(1000)\cos 40°$
$W \approx 459.6266659$
459.63 W

57. $F_N - mg\cos\theta = 0$
$F_N = mg\cos\theta$
$mg\sin\theta - \mu_k F_N = 0$
$mg\sin\theta - \mu_k(mg\cos\theta) = 0$
$\mu_k(mg\cos\theta) = mg\sin\theta$
$\mu_k = \frac{mg\sin\theta}{mg\cos\theta}$
$\mu_k = \frac{\sin\theta}{\cos\theta}$
$\mu_k = \tan\theta$

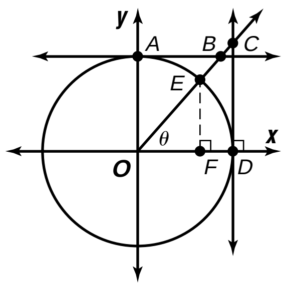
58.



$\theta = \frac{360°}{2n} = \frac{180°}{n}$, $\tan\theta = \frac{\frac{a}{2}}{h}$, so $h = \frac{a}{2\tan\theta} = \frac{a}{2}\cot\theta$.

The area of the isosceles triangle is $\frac{1}{2}(a)\left(\frac{a}{2}\cot\frac{180°}{n}\right)$ $= \frac{a^2}{4}\cot\left(\frac{180°}{n}\right)$. There are n such triangles, so $A = \frac{1}{4}na^2\cot\left(\frac{180°}{n}\right)$.

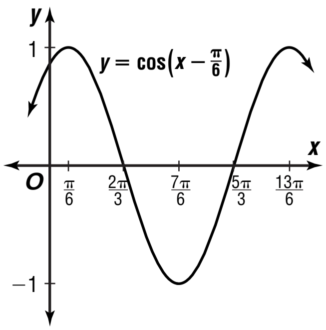
59.



$\sin\theta = EF$ and $\cos\theta = OF$ since the circle is a unit circle. $\tan\theta = \frac{CD}{OD} = \frac{CD}{1} = CD$.
$\sec\theta = \frac{CO}{OD} = \frac{CO}{1} = CO$. $\triangle EOF \sim \triangle OBA$, so $\frac{OF}{EF} = \frac{BA}{OA} = \frac{BA}{1} = BA$. Then $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{OF}{EF} = BA$.
Also by similar triangles, $\frac{EO}{EF} = \frac{OB}{OA}$, or $\frac{1}{EF} = \frac{OB}{1}$.
Then $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{EF} = \frac{OB}{1} = OB$.

60. $\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 135°$

61.



62. $2(3° \ 30') = 7°$ $\qquad 7° = 7° \times \frac{\pi}{180°}$

$= \frac{7\pi}{180}$

$s = r\theta$

$s = 20\left(\frac{7\pi}{180}\right)$

$s \approx 2.44$ cm

63. $B = 180° - (90° + 20°)$ or $70°$

$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c}$

$\sin 20° = \frac{a}{35} \qquad \cos 20° = \frac{b}{35}$

$35 \sin 20° = a \qquad 35 \cos 20° = b$

$11.97070502 \approx a \qquad 32.88924173 \approx b$

$a = 12.0, B = 70°, b = 32.9$

64.
$$\begin{array}{r|rrrr} 2 & 2 & 1 & -8 & -4 \\ & & 4 & 10 & 4 \\ \hline & 2 & 5 & 2 & 0 \end{array}$$

$2x^2 + 5x + 2 = 0$

$(2x + 1)(x + 2) = 0$

$2x + 1 = 0$ or $x + 2 = 0$

$x = -\frac{1}{2} \qquad x = -2$

$-2, -\frac{1}{2}, 2$

65. $2x^2 + 7x - 4 = 0$

$x^2 + \frac{7}{2}x - 2 = 0$

$x^2 + \frac{7}{2}x = 2$

$x^2 + \frac{7}{2}x + \frac{49}{16} = 2 + \frac{49}{16}$

$\left(x + \frac{7}{4}\right)^2 = \frac{81}{16}$

$x + \frac{7}{4} = \pm\frac{9}{4}$

$x = -\frac{7}{4} \pm \frac{9}{4}$

$x = 0.5$ or -4

66. continuous

67. $4(x + y - 2z) = 4(3) \qquad\qquad 4x + 4y - 8z = 12$

$-4x - y - z = 0 \quad \rightarrow \quad \underline{-4x - y - z = 0}$

$3y - 9z = 12$

$x + y - 2z = 3$

$\underline{-x - 5y + 4z = 11}$

$-4y + 2z = 14$

$4(3y - 9z) = 4(12) \quad \rightarrow \quad 12y - 36z = 48$

$3(-4y + 2z) = 3(14) \qquad \underline{-12y + 6z = 42}$

$-30z = 90$

$z = -3$

$3y - 9z = 12 \qquad x + y - 2z = 3$

$3y - 9(-3) = 12 \qquad x + (-5) - 2(-3) = 3$

$y = -5 \qquad x = 2$

$(2, -5, -3)$

68. $m = \frac{4 - 2}{-4 - 5}$

$= \frac{2}{-9}$ or $-\frac{2}{9}$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{2}{9}(x - (-4))$

$y = -\frac{2}{9}x + \frac{28}{9}$

69. $m\angle BCD = 40°$

$40 = \frac{1}{2}m(\widehat{BC})$

$80 = m(\widehat{BC})$

$m\angle BAC = \frac{1}{2}m\widehat{BC}$

$m\angle BAC = \frac{1}{2}(80)$

$m\angle BAC = 40°$

The correct choice is C.

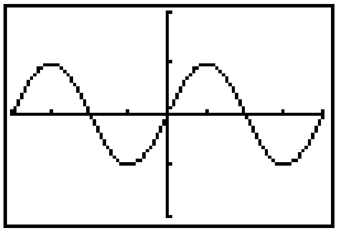
7-2 Verifying Trigonometric Identities

Page 433 Graphing Calculator Exploration

1. yes **2.** no **3.** no

4. No; it is impossible to look at every window since there are an infinite number. The only way an identity can be proven is by showing algebraically that the general case is true.

5.



$[-2\pi, 2\pi]$ scl:$\frac{\pi}{2}$ by $[-2, 2]$ scl:1

$\sin x$

Pages 433–434 Check for Understanding

1. Answers will vary.

2. Sample answer: Squaring each side can turn two unequal quantities into equal quantities. For example, $-1 \neq 1$, but $(-1)^2 = 1^2$.

3. Sample answer: They are the trigonometric functions with which most people are most familiar.

4. Answers will vary.

5. $\cos x \stackrel{?}{=} \frac{\cot x}{\csc x}$

$\cos x \stackrel{?}{=} \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$

$\cos x \stackrel{?}{=} \frac{\cos x}{1}$

$\cos x = \cos x$

6. $\frac{1}{\tan x + \sec x} \stackrel{?}{=} \frac{\cos x}{\sin x + 1}$

$\frac{1}{\frac{\sin x}{\cos x} + \frac{1}{\cos x}} \stackrel{?}{=} \frac{\cos x}{\sin x + 1}$

$\frac{1}{\frac{\sin x + 1}{\cos x}} \stackrel{?}{=} \frac{\cos x}{\sin x + 1}$

$\frac{\cos x}{\sin x + 1} = \frac{\cos x}{\sin x + 1}$

7. $\csc\theta - \cot\theta \stackrel{?}{=} \frac{1}{\csc\theta + \cot\theta}$

$\csc\theta - \cot\theta \stackrel{?}{=} \frac{1}{\csc\theta + \cot\theta} \cdot \frac{\csc\theta - \cot\theta}{\csc\theta - \cot\theta}$

$\csc\theta - \cot\theta \stackrel{?}{=} \frac{\csc\theta - \cot\theta}{\csc^2\theta - \cot^2\theta}$

$\csc\theta - \cot\theta \stackrel{?}{=} \frac{\csc\theta - \cot\theta}{(1 + \cot^2\theta) - \cot^2\theta}$

$\csc\theta - \cot\theta \stackrel{?}{=} \frac{\csc\theta - \cot\theta}{1}$

$\csc\theta - \cot\theta = \csc\theta - \cot\theta$

8. $\sin\theta\tan\theta \stackrel{?}{=} \sec\theta - \cos\theta$

$\sin\theta\tan\theta \stackrel{?}{=} \frac{1}{\cos\theta} - \cos\theta$

$\sin\theta\tan\theta \stackrel{?}{=} \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$

$\sin\theta\tan\theta \stackrel{?}{=} \frac{1 - \cos^2\theta}{\cos\theta}$

$\sin\theta\tan\theta \stackrel{?}{=} \frac{\sin^2\theta}{\cos\theta}$

$\sin\theta\tan\theta \stackrel{?}{=} \sin\theta\frac{\sin\theta}{\cos\theta}$

$\sin\theta\tan\theta = \sin\theta\tan\theta$

9. $(\sin A - \cos A)^2 \stackrel{?}{=} 1 - 2\sin^2 A\cot A$

$\sin^2 A - 2\sin A\cos A + \cos^2 A \stackrel{?}{=} 1 - 2\sin^2 A\cot A$

$1 - 2\sin A\cos A \stackrel{?}{=} 1 - 2\sin^2 A\cot A$

$1 - 2\sin A\cos A\frac{\sin A}{\sin A} \stackrel{?}{=} 1 - 2\sin^2 A\cot A$

$1 - 2\sin^2 A\frac{\cos A}{\sin A} \stackrel{?}{=} 1 - 2\sin^2 A\cot A$

$1 - 2\sin^2 A\cot A = 1 - 2\sin^2 A\cot A$

10. Sample answer: $\sin x = \frac{1}{4}$

$\tan x = \frac{1}{4}\sec x$

$\frac{\tan x}{\sec x} = \frac{1}{4}$

$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{1}{4}$

$\sin x = \frac{1}{4}$

11. Sample answer: $\cos x = -1$

$\cot x + \sin x = -\cos x\cot x$

$\frac{\cos x}{\sin x} + \sin x = -\cos x\frac{\cos x}{\sin x}$

$\cos x + \sin^2 x = -\cos^2 x$

$\cos^2 x + \sin^2 x = -\cos x$

$1 = -\cos x$

$\cos x = -1$

12. $\frac{I\cos\theta}{R^2} \stackrel{?}{=} \frac{I\cot\theta}{R^2\csc\theta}$

$\frac{I\cos\theta}{R^2} \stackrel{?}{=} \frac{I\frac{\cos\theta}{\sin\theta}}{R^2\frac{1}{\sin\theta}}$

$\frac{I\cos\theta}{R^2} \stackrel{?}{=} \frac{I\frac{\cos\theta}{\sin\theta}}{R^2\frac{1}{\sin\theta}} \cdot \frac{\sin\theta}{\sin\theta}$

$\frac{I\cos\theta}{R^2} = \frac{I\cos\theta}{R^2}$

Pages 434–436 Exercises

13. $\tan A \stackrel{?}{=} \frac{\sec A}{\csc A}$

$\tan A \stackrel{?}{=} \frac{\frac{1}{\cos A}}{\frac{1}{\sin A}}$

$\tan A \stackrel{?}{=} \frac{\sin A}{\cos A}$

$\tan A = \tan A$

14. $\cos\theta \stackrel{?}{=} \sin\theta\cot\theta$

$\cos\theta \stackrel{?}{=} \sin\theta\frac{\cos\theta}{\sin\theta}$

$\cos\theta = \cos\theta$

15. $\sec x - \tan x = \frac{1 - \sin x}{\cos x}$

$\sec x - \tan x \stackrel{?}{=} \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$\sec x - \tan x = \sec x - \tan x$

16. $\frac{1 + \tan x}{\sin x + \cos x} \stackrel{?}{=} \sec x$

$\frac{1 + \frac{\sin x}{\cos x}}{\sin x + \cos x} \stackrel{?}{=} \sec x$

$\frac{\cos x}{\cos x} \cdot \frac{1 + \frac{\sin x}{\cos x}}{\sin x + \cos x} \stackrel{?}{=} \sec x$

$\frac{\cos x + \sin x}{\cos x(\sin x + \cos x)} \stackrel{?}{=} \sec x$

$\frac{1}{\cos x} \stackrel{?}{=} \sec x$

$\sec x = \sec x$

17. $\sec x\csc x \stackrel{?}{=} \tan x + \cot x$

$\sec x\csc x \stackrel{?}{=} \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$\sec x\csc x \stackrel{?}{=} \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$

$\sec x\csc x \stackrel{?}{=} \frac{\sin^2 x}{\cos x\sin x} + \frac{\cos^2 x}{\sin x\cos x}$

$\sec x\csc x \stackrel{?}{=} \frac{\sin^2 x + \cos^2 x}{\cos x\sin x}$

$\sec x\csc x \stackrel{?}{=} \frac{1}{\cos x\sin x}$

$\sec x\csc x \stackrel{?}{=} \frac{1}{\cos x} \cdot \frac{1}{\sin x}$

$\sec x\csc x = \sec x\csc x$

18. $\sin\theta + \cos\theta \stackrel{?}{=} \frac{2\sin^2\theta - 1}{\sin\theta - \cos\theta}$

$\sin\theta + \cos\theta \stackrel{?}{=} \frac{2\sin^2\theta - (\sin^2\theta + \cos^2\theta)}{\sin\theta - \cos\theta}$

$\sin\theta + \cos\theta \stackrel{?}{=} \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$

$\sin\theta + \cos\theta \stackrel{?}{=} \frac{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}{\sin\theta - \cos\theta}$

$\sin\theta + \cos\theta = \sin\theta + \cos\theta$

19. $(\sin A + \cos A)^2 \stackrel{?}{=} \frac{2 + \sec A\csc A}{\sec A\csc A}$

$(\sin A + \cos A)^2 \stackrel{?}{=} \frac{2}{\sec A\csc A} + \frac{\sec A\csc A}{\sec A\csc A}$

$(\sin A + \cos A)^2 \stackrel{?}{=} 2\frac{1}{\sec A} \cdot \frac{1}{\csc A} + 1$

$(\sin A + \cos A)^2 \stackrel{?}{=} 2\cos A\sin A + 1$

$(\sin A + \cos A)^2 \stackrel{?}{=} 2\cos A\sin A + \sin^2 A + \cos^2 A$

$(\sin A + \cos A)^2 = (\sin A + \cos A)^2$

20. $$(\sin\theta - 1)(\tan\theta + \sec\theta) \stackrel{?}{=} -\cos\theta$$
$$\sin\theta\tan\theta - \tan\theta + \sin\theta\sec\theta - \sec\theta \stackrel{?}{=} -\cos\theta$$
$$\sin\theta\frac{\sin\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} + \sin\theta\frac{1}{\cos\theta} - \frac{1}{\cos\theta} \stackrel{?}{=} -\cos\theta$$
$$\frac{\sin^2\theta - \sin\theta + \sin\theta - 1}{\cos\theta} \stackrel{?}{=} -\cos\theta$$
$$\frac{\sin^2\theta - 1}{\cos\theta} \stackrel{?}{=} -\cos\theta$$
$$\frac{-\cos^2\theta}{\cos\theta} \stackrel{?}{=} -\cos\theta$$
$$-\cos\theta = -\cos\theta$$

21. $$\frac{\cos y}{1-\sin y} \stackrel{?}{=} \frac{1+\sin y}{\cos y}$$
$$\frac{\cos y}{1-\sin y}\cdot\frac{1+\sin y}{1+\sin y} \stackrel{?}{=} \frac{1+\sin y}{\cos y}$$
$$\frac{\cos y(1-\sin y)}{1-\sin^2 y} \stackrel{?}{=} \frac{1+\sin y}{\cos y}$$
$$\frac{\cos y(1+\sin y)}{\cos^2 y} \stackrel{?}{=} \frac{1+\sin y}{\cos y}$$
$$\frac{1+\sin y}{\cos y} = \frac{1+\sin y}{\cos y}$$

22. $$\cos\theta\cos(-\theta) - \sin\theta\sin(-\theta) \stackrel{?}{=} 1$$
$$\cos\theta\cos\theta - \sin\theta(-\sin\theta) \stackrel{?}{=} 1$$
$$\cos^2\theta + \sin^2\theta \stackrel{?}{=} 1$$
$$1 = 1$$

23. $$\csc x - 1 \stackrel{?}{=} \frac{\cot^2 x}{\csc x + 1}$$
$$\csc x - 1 \stackrel{?}{=} \frac{\csc^2 x - 1}{\csc x + 1}$$
$$\csc x - 1 \stackrel{?}{=} \frac{(\csc x + 1)(\csc x - 1)}{\csc x + 1}$$
$$\csc x - 1 = \csc x - 1$$

24. $$\cos B\cot B \stackrel{?}{=} \csc B - \sin B$$
$$\cos B\cot B \stackrel{?}{=} \frac{1}{\sin B} - \sin B$$
$$\cos B\cot B \stackrel{?}{=} \frac{1}{\sin B} - \frac{\sin^2 B}{\sin B}$$
$$\cos B\cot B \stackrel{?}{=} \frac{1-\sin^2 B}{\sin B}$$
$$\cos B\cot B \stackrel{?}{=} \frac{\cos^2 B}{\sin B}$$
$$\cos B\cot B \stackrel{?}{=} \cos B\frac{\cos B}{\sin B}$$
$$\cos B\cot B = \cos B\cot B$$

25. $$\sin\theta\cos\theta\tan\theta + \cos^2\theta \stackrel{?}{=} 1$$
$$\sin\theta\cos\theta\frac{\sin\theta}{\cos\theta} + \cos^2\theta \stackrel{?}{=} 1$$
$$\sin^2\theta + \cos^2\theta \stackrel{?}{=} 1$$
$$1 = 1$$

26. $$(\csc x - \cot x)^2 \stackrel{?}{=} \frac{1-\cos x}{1+\cos x}$$
$$\csc^2 x - 2\csc x\cot x + \cot^2 x \stackrel{?}{=} \frac{1-\cos x}{1+\cos x}$$
$$\frac{1}{\sin^2 x} - 2\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x} + \frac{\cos^2 x}{\sin^2 x} \stackrel{?}{=} \frac{1-\cos x}{1+\cos x}$$
$$\frac{1 - 2\cos x + \cos^2 x}{\sin^2 x} \stackrel{?}{=} \frac{1-\cos x}{1+\cos x}$$
$$\frac{(1-\cos x)^2}{1-\cos^2 x} \stackrel{?}{=} \frac{1-\cos x}{1+\cos x}$$
$$\frac{(1-\cos x)^2}{(1-\cos x)(1+\cos x)} \stackrel{?}{=} \frac{1-\cos x}{1+\cos x}$$
$$\frac{1-\cos x}{1+\cos x} = \frac{1-\cos x}{1+\cos x}$$

27. $$\sin x + \cos x \stackrel{?}{=} \frac{\cos x}{1-\tan x} + \frac{\sin x}{1-\cot x}$$
$$\sin x + \cos x \stackrel{?}{=} \frac{\cos x}{1-\frac{\sin x}{\cos x}} + \frac{\sin x}{1-\frac{\cos x}{\sin x}}$$
$$\sin x + \cos x \stackrel{?}{=} \frac{\cos x}{1-\frac{\sin x}{\cos x}}\cdot\frac{\cos x}{\cos x} + \frac{\sin x}{1-\frac{\cos x}{\sin x}}\cdot\frac{\sin x}{\sin x}$$
$$\sin x + \cos x \stackrel{?}{=} \frac{\cos^2 x}{\cos x - \sin x} + \frac{\sin^2 x}{\sin x - \cos x}$$
$$\sin x + \cos x \stackrel{?}{=} -\frac{\cos^2 x}{\sin x - \cos x} + \frac{\sin^2 x}{\sin x - \cos x}$$
$$\sin x + \cos x \stackrel{?}{=} \frac{\sin^2 x - \cos^2 x}{\sin x - \cos x}$$
$$\sin x + \cos x \stackrel{?}{=} \frac{(\sin x + \cos x)(\sin x - \cos x)}{\sin x - \cos x}$$
$$\sin x + \cos x = \sin x + \cos x$$

28. $$\sin\theta + \cos\theta + \tan\theta\sin\theta \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\sin\theta + \cos\theta + \frac{\sin\theta}{\cos\theta}\sin\theta \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\sin\theta + \cos\theta + \frac{\sin^2\theta}{\cos\theta} \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\sin\theta + \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta} \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\sin\theta + \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\sin\theta + \frac{1}{\cos\theta} \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\sin\theta + \sec\theta \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\sin\theta\frac{\cos\theta}{\cos\theta} + \sec\theta \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\cos\theta\frac{\sin\theta}{\cos\theta} + \sec\theta \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\cos\theta\tan\theta + \sec\theta \stackrel{?}{=} \sec\theta + \cos\theta\tan\theta$$
$$\sec\theta + \cos\theta\tan\theta = \sec\theta + \cos\theta\tan\theta$$

29. Sample answer: $\sec x = \sqrt{2}$
$$\frac{\csc x}{\cot x} = \sqrt{2}$$
$$\frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \sqrt{2}$$
$$\frac{1}{\cos x} = \sqrt{2}$$
$$\sec x = \sqrt{2}$$

30. Sample answer: $\tan x = 2$
$$\frac{1+\tan x}{1+\cot x} = 2$$
$$\frac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}} = 2$$
$$\frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}} = 2$$
$$\frac{\sin x}{\cos x} = 2$$
$$\tan x = 2$$

31. Sample answer: $\cos x = 0$
$$\frac{1}{\cot x} - \frac{\sec x}{\csc x} = \cos x$$
$$\tan x - \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \cos x$$
$$\tan x - \frac{\sin x}{\cos x} = \cos x$$
$$\tan x - \tan x = \cos x$$
$$0 = \cos x$$

32. Sample answer: $\sin x = \frac{1}{2}$

$$\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x} = 4$$
$$\frac{1+2\cos x+\cos^2 x}{\sin x(1+\cos x)} + \frac{\sin^2 x}{\sin x(1+\cos x)} = 4$$
$$\frac{1+2\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)} = 4$$
$$\frac{2+2\cos x}{\sin x(1+\cos x)} = 4$$
$$\frac{2(1+\cos x)}{\sin x(1+\cos x)} = 4$$
$$\frac{2}{\sin x} = 4$$
$$2 = 4\sin x$$
$$\frac{1}{2} = \sin x$$

33. Sample answer: $\sin x = 1$

$$\cos^2 x + 2\sin x - 2 = 0$$
$$1 - \sin^2 x + 2\sin x - 2 = 0$$
$$0 = \sin^2 x - 2\sin x + 1$$
$$0 = (\sin x - 1)^2$$
$$0 = \sin x - 1$$
$$\sin x = 1$$

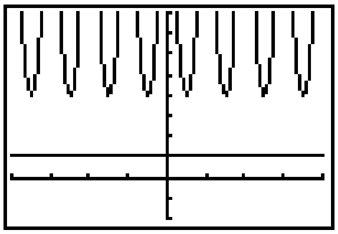
34. Sample answer: $\cot x = 1$

$$\csc x = \sin x \tan x + \cos x$$
$$\csc x = \sin x \frac{\sin x}{\cos x} + \cos x$$
$$\csc x = \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$
$$\csc x = \frac{1}{\cos x}$$
$$\frac{1}{\sin x} = \frac{1}{\cos x}$$
$$\frac{\cos x}{\sin x} = 1$$
$$\cot x = 1$$

35.
$$\frac{\tan^3\theta - 1}{\tan\theta - 1} - \sec^2\theta - 1 = 0$$
$$\frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta-1} - (\tan^2\theta+1) - 1 = 0$$
$$\tan^2\theta + \tan\theta + 1 - \tan^2\theta - 1 - 1 = 0$$
$$\tan\theta - 1 = 0$$
$$\tan\theta = 1$$

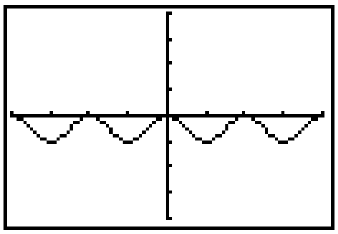
$$\cot\theta = \frac{1}{\tan\theta}$$
$$\cot\theta = \frac{1}{1}$$
$$\cot\theta = 1$$

36. no



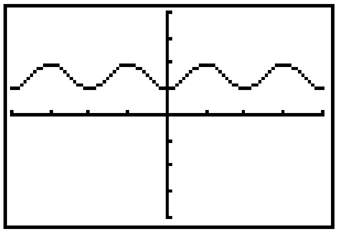
$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-2, 8]$ sc1:1

37. yes



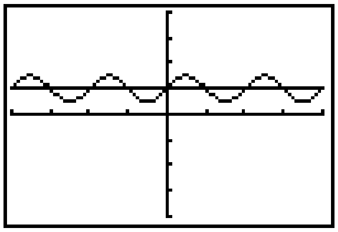
$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-4, 4]$ sc1:1

38. yes



$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-4, 4]$ sc1:1

39. no



$[-2\pi, 2\pi]$ sc1:$\frac{\pi}{2}$ by $[-4, 4]$ sc1:1

40a. $P = I_0^2 R \sin^2 2\pi ft$

$P = I_0^2 R(1 - \cos^2 2\pi ft)$

40b. $P = I_0^2 R \sin^2 2\pi ft$

$P = \frac{I_0^2 R}{\csc^2 2\pi ft}$

41.
$$f(x) = \frac{x}{\sqrt{1+4x^2}}$$
$$f(x) = \frac{\frac{1}{2}\tan\theta}{\sqrt{1+4\left(\frac{1}{2}\tan\theta\right)^2}}$$
$$f(x) = \frac{\frac{1}{2}\tan\theta}{\sqrt{1+\tan^2\theta}}$$
$$f(x) = \frac{\frac{1}{2}\tan\theta}{\sqrt{\sec^2\theta}}$$
$$f(x) = \frac{\frac{1}{2}\tan\theta}{\sec\theta}$$
$$f(x) = \frac{\frac{1}{2}\cdot\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$
$$f(x) = \frac{1}{2}\sin\theta$$

42. $\sin a = \sin\alpha \sin c \Rightarrow \sin\alpha = \frac{\sin a}{\sin c}$

$\cos b = \frac{\cos\beta}{\sin\alpha} \Rightarrow \cos\beta = \sin\alpha\cos b$

$\cos c = \cos a\cos b \Rightarrow \cos b = \frac{\cos c}{\cos a}$

Then $\cos\beta = \sin\alpha\cos b$
$$= \frac{\sin a}{\sin c}\cdot\frac{\cos c}{\cos a}$$
$$= \frac{\sin a}{\cos a}\cdot\frac{\cos c}{\sin c}$$
$$= \tan a \cot c$$

43.
$$y = \frac{-gv^2}{2v_0^2\cos^2\theta} + \frac{x\sin\theta}{\cos\theta}$$
$$y = \frac{-gv^2}{2v_0^2}\sec^2\theta + x\tan\theta$$
$$y = -\frac{gx^2}{2v_0^2}(1+\tan^2\theta) + x\tan\theta$$

44. We find the area of $ABTP$ by subtracting the area of $\triangle OAP$ from the area of $\triangle OBT$.

$$\frac{1}{2}OB \cdot BT - \frac{1}{2}OA \cdot AP = \frac{1}{2} \cdot 1 \cdot \tan\theta - \frac{1}{2}\cos\theta\sin\theta$$
$$= \frac{1}{2}\left(\frac{\sin\theta}{\cos\theta} - \cos\theta\sin\theta\right)$$
$$= \frac{1}{2}\sin\theta\left(\frac{1}{\cos\theta} - \cos\theta\right)$$
$$= \frac{1}{2}\sin\theta\left(\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}\right)$$
$$= \frac{1}{2}\sin\theta\left(\frac{1-\cos^2\theta}{\cos\theta}\right)$$
$$= \frac{1}{2}\sin\theta\left(\frac{\sin^2\theta}{\cos\theta}\right)$$
$$= \frac{1}{2} \cdot \frac{\sin\theta}{\cos\theta}\sin^2\theta$$
$$= \frac{1}{2}\tan\theta\sin^2\theta$$

45. By the Law of Sines, $\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$, so $b = \frac{a\sin\beta}{\sin\alpha}$. Then

$$A = \frac{1}{2}ab\sin\gamma$$
$$A = \frac{1}{2}a\left(\frac{a\sin\beta}{\sin\alpha}\right)\sin\gamma$$
$$A = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$$
$$A = \frac{a^2\sin\beta\sin\gamma}{2\sin(180° - (\beta+\gamma))}$$
$$A = \frac{a^2\sin\beta\sin\gamma}{2\sin(\beta+\gamma)}$$

46. $$\frac{\tan x + \cos x + \sin x\tan x}{\sec x + \tan x} = \frac{\frac{\sin x}{\cos x} + \cos x + \sin x\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}$$
$$= \frac{\frac{\sin x + \cos^2 x + \sin^2 x}{\cos x}}{\frac{1+\sin x}{\cos x}}$$
$$= \frac{\sin x + 1}{\cos x} \cdot \frac{\cos x}{1+\sin x}$$
$$= 1$$

47. $|A| = 2$ $\qquad \frac{360°}{k} = 180° \qquad -\frac{c}{2} = 45°$

$A = \pm 2 \qquad k = 2 \qquad c = -90°$

$y = \pm 2\sin(2x - 90°)$

48. $\frac{15\pi}{16} = \frac{15\pi}{16} \times \frac{180°}{\pi}$

$= 168.75°$

$168.75° = 168° + \left(0.75° \times \frac{60'}{1°}\right)$

$= 168° + 45'$

$168°\ 45'$

49. $\sqrt[3]{3y-1} - 2 = 0$ $\qquad$ Check: $\sqrt[3]{3y-1} - 2 = 0$

$\sqrt[3]{3y-1} = 2 \qquad \sqrt[3]{3(3)-1} - 2 \stackrel{?}{=} 0$

$3y - 1 = 8 \qquad \sqrt[3]{8} - 2 \stackrel{?}{=} 0$

$y = 3 \qquad 2 - 2 = 0\ \checkmark$

50. $x + 1 = 0$

$x = -1$

$f(x) = \frac{3x}{x+1}$

$y = \frac{3x}{x+1}$

$y(x+1) = 3x$

$yx + y = 3x$

$y = 3x - yx$

$y = x(3-y)$

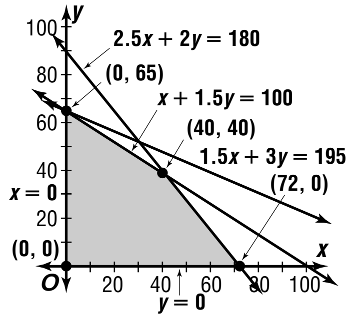
$\frac{y}{3-y} = x$

$3 - y = 0$

$y = 3$

51. Let x = the number of shirts and y = the number of pants.

$x + 1.5y \le 100$
$2.5x + 2y \le 180$
$1.5x + 3y \le 195$
$x \ge 0$
$y \ge 0$



$P(x, y) = 5x + 4.5y$
$P(0, 0) = 5(0) + 4.5(0)$ or 0
$P(0, 65) = 5(0) + 4.5(65)$ or 292.50
$P(40, 40) = 5(40) + 4.5(40)$ or 380
$P(72, 0) = 5(72) + 4.5(0)$ or 360

40 shirts, 40 pants

52. {16}, {−4, 4}; no, 16 is paired with two elements of the range

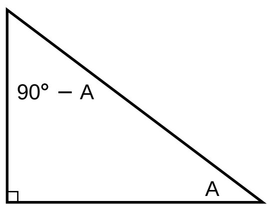
53. $$\frac{a-b}{a+b} \div \frac{b-a}{b+a} = \frac{a-b}{a+b} \cdot \frac{b+a}{b-a}$$
$$= \frac{a-b}{a+b} \cdot \frac{a+b}{-1(a-b)}$$
$$= -1$$

The correct choice is D.

7-3 Sum and Difference Identities

Pages 441–442 Check for Understanding

1. Find a counterexample, such as $x = 30°$ and $y = 60°$.
2. Find the cosine, sine, or tangent, respectively, of the sum or difference, then take the reciprocal.
3. The opposite side for $90° - A$ is the adjacent side for A, so the right-triangle ratio for $\sin(90° - A)$ is the same as that for $\cos A$.



4. $$\cot(\alpha+\beta) = \frac{1}{\tan(\alpha+\beta)}$$
$$= \frac{1}{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$
$$= \frac{1 - \tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$$
$$= \frac{1 - \frac{1}{\cot\alpha}\cdot\frac{1}{\cot\beta}}{\frac{1}{\cot\alpha} + \frac{1}{\cot\beta}} \cdot \frac{\cot\alpha\cot\beta}{\cot\alpha\cot\beta}$$
$$= \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$$

5. $\cos 165° = \cos(45° + 120°)$
$= \cos 45° \cos 120° - \sin 45° \sin 120°$
$= \frac{\sqrt{2}}{2} \cdot \left(-\frac{1}{2}\right) - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$
$= -\frac{\sqrt{2} + \sqrt{6}}{4}$

6. $\tan \frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$
$= \dfrac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{4}}$
$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1}$
$= \frac{-4 + 2\sqrt{3}}{-2}$
$= 2 - \sqrt{3}$

7. $795° = 2(360°) + 75°$
$\sec 795° = \sec 75°$
$\cos 75° = \cos(30° + 45°)$
$= \cos 30° \cos 45° - \sin 30° \sin 45°$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$
$\sec 795° = \frac{4}{\sqrt{6} - \sqrt{2}}$
$= \sqrt{6} + \sqrt{2}$

8. $\cos x = \sqrt{1 - \sin^2 x}$
$= \sqrt{1 - \left(\frac{4}{9}\right)^2}$
$= \sqrt{\frac{65}{81}}$ or $\frac{\sqrt{65}}{9}$

$\cos y = \sqrt{1 - \sin^2 y}$
$= \sqrt{1 - \left(\frac{1}{4}\right)^2}$
$= \sqrt{\frac{15}{16}}$ or $\frac{\sqrt{15}}{4}$

$\sin(x - y) = \sin x \cos y - \cos x \sin y$
$= \left(\frac{4}{9}\right)\left(\frac{\sqrt{15}}{4}\right) - \left(\frac{\sqrt{65}}{9}\right)\left(\frac{1}{4}\right)$
$= \frac{4\sqrt{15} - \sqrt{65}}{36}$

9. $\csc x = \frac{1}{\sin x}$
$\frac{5}{3} = \frac{1}{\sin x}$
$\sin x = \frac{3}{5}$
$\tan x = \frac{\sin x}{\cos x}$
$= \dfrac{\frac{3}{5}}{\frac{4}{5}}$ or $\frac{3}{4}$

$\cos x = \sqrt{1 - \sin^2 x}$
$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$
$= \sqrt{\frac{16}{25}}$ or $\frac{4}{5}$

$\sin y = \sqrt{1 - \cos^2 y}$
$= \sqrt{1 - \left(\frac{5}{13}\right)^2}$
$= \sqrt{\frac{144}{169}}$ or $\frac{12}{13}$

$\tan y = \frac{\sin y}{\cos y}$
$= \dfrac{\frac{12}{13}}{\frac{5}{13}}$ or $\frac{12}{5}$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
$= \dfrac{\frac{3}{4} + \frac{12}{5}}{1 - \left(\frac{3}{4}\right)\left(\frac{12}{5}\right)}$
$= \dfrac{\frac{63}{20}}{-\frac{4}{5}}$
$= -\frac{63}{16}$

10. $\sin(90° + A) \stackrel{?}{=} \cos A$
$\sin 90° \cos A + \cos 90° \sin A \stackrel{?}{=} \cos A$
$1 \cdot \cos A + 0 \cdot \sin A \stackrel{?}{=} \cos A$
$\cos A = \cos A$

11. $\tan\left(\theta + \frac{\pi}{2}\right) \stackrel{?}{=} -\cot \theta$
$\dfrac{\sin\left(\theta + \frac{\pi}{2}\right)}{\cos\left(\theta + \frac{\pi}{2}\right)} \stackrel{?}{=} -\cot \theta$
$\dfrac{\sin \theta \cos \frac{\pi}{2} + \cos \theta \sin \frac{\pi}{2}}{\cos \theta \cos \frac{\pi}{2} - \sin \theta \sin \frac{\pi}{2}} \stackrel{?}{=} -\cot \theta$
$\frac{(\sin \theta) \cdot 0 + (\cos \theta) \cdot 1}{(\cos \theta) \cdot 0 - (\sin \theta) \cdot 1} \stackrel{?}{=} -\cot \theta$
$-\frac{\cos \theta}{\sin \theta} \stackrel{?}{=} -\cot \theta$
$-\cot \theta = -\cot \theta$

12. $\sin(x - y) \stackrel{?}{=} \frac{1 - \cot x \tan y}{\csc x \sec y}$
$\sin(x - y) \stackrel{?}{=} \dfrac{1 - \frac{\cos x}{\sin x} \cdot \frac{\sin y}{\cos y}}{\frac{1}{\sin x} \cdot \frac{1}{\cos y}}$
$\sin(x - y) \stackrel{?}{=} \dfrac{1 - \frac{\cos x}{\sin x} \cdot \frac{\sin y}{\cos y}}{\frac{1}{\sin x} \cdot \frac{1}{\cos y}} \cdot \frac{\sin x \cos y}{\sin x \cos y}$
$\sin(x - y) \stackrel{?}{=} \frac{\sin x \cos y - \cos x \sin y}{1}$
$\sin(x - y) = \sin(x - y)$

13. $\sin(n\omega_0 t - 90°) = \sin n\omega_0 t \cos 90° - \cos n\omega_0 t \sin 90°$
$= \sin n\omega_0 t \cdot 0 - \cos n\omega_0 t \cdot 1$
$= -\cos n\omega_0 t$

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14. $\cos 105° = \cos(45° + 60°)$
$= \cos 45° \cos 60° - \sin 45° \sin 60°$
$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$
$= \frac{\sqrt{2} - \sqrt{6}}{4}$

15. $\sin 165° = \sin(120° + 45°)$
$= \sin 120° \cos 45° + \cos 120° \sin 45°$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

16. $\cos \frac{7\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$
$= \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3}$
$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$
$= \frac{\sqrt{2} - \sqrt{6}}{4}$

17. $\sin \frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$
$= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

18. $\tan 195° = \tan(45° + 150°)$

$= \frac{\tan 45° + \tan 150°}{1 - \tan 45° \tan 150°}$

$= \frac{1 + \left(-\frac{\sqrt{3}}{3}\right)}{1 - 1\left(-\frac{\sqrt{3}}{3}\right)}$

$= \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$

$= \frac{12 - 6\sqrt{3}}{6}$ or $2 - \sqrt{3}$

19. $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

20. $\tan 165° = \tan(45° + 120°)$

$= \frac{\tan 45° + \tan 120°}{1 - \tan 45° \tan 120°}$

$= \frac{1 + (-\sqrt{3})}{1 - 1(-\sqrt{3})}$

$= \frac{1-\sqrt{3}}{1+\sqrt{3}}$

$= \frac{4 - 2\sqrt{3}}{-2}$ or $-2 + \sqrt{3}$

21. $\tan\frac{23\pi}{12} = \tan\left(\frac{\pi}{4} + \frac{5\pi}{3}\right)$

$= \frac{\tan\frac{\pi}{4} + \tan\frac{5\pi}{3}}{1 - \tan\frac{\pi}{4}\tan\frac{5\pi}{3}}$

$= \frac{1 + (-\sqrt{3})}{1 - 1(-\sqrt{3})}$

$= \frac{4 - 2\sqrt{3}}{-2}$ or $-2 + \sqrt{3}$

22. $735° = 2(360°) + 15°$

$\sin 735° = \sin 15°$

$\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

23. $1275° = 3(360°) + 195°$

$\sec 1275° = \sec 195°$

$\cos 195° = \cos(150° + 45°)$

$= \cos 150° \cos 45° - \sin 150° \sin 45°$

$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{6}-\sqrt{2}}{4}$

$\sec 1275° = \frac{4}{-\sqrt{6}-\sqrt{2}}$

$= \sqrt{2} - \sqrt{6}$

24. $\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

$\csc\frac{5\pi}{2} = \frac{4}{\sqrt{2}+\sqrt{6}}$

$= \sqrt{6} - \sqrt{2}$

25. $\frac{113\pi}{12} = 4(2\pi) + \frac{17\pi}{12}$

$\cot\frac{113\pi}{12} = \cot\frac{17\pi}{12}$

$\tan\frac{17\pi}{12} = \tan\left(\frac{7\pi}{6} + \frac{\pi}{4}\right)$

$= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}$

$= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}\cdot 1}$

$= \frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}}$

$= \sqrt{3} + 2$

$\cot\frac{113\pi}{12} = \frac{1}{\sqrt{3}+2}$

$= 2 - \sqrt{3}$

26. $\sin x = \sqrt{1 - \cos^2 x}$

$= \sqrt{1 - \left(\frac{8}{17}\right)^2}$

$= \sqrt{\frac{225}{289}}$ or $\frac{15}{17}$

$\cos y = \sqrt{1 - \sin^2 y}$

$= \sqrt{1 - \left(\frac{12}{37}\right)^2}$

$= \sqrt{\frac{1225}{1369}}$ or $\frac{35}{37}$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{15}{17}\right)\left(\frac{35}{37}\right) + \left(\frac{8}{17}\right)\left(\frac{12}{37}\right)$

$= \frac{621}{629}$

27. $\sin x = \sqrt{1 - \cos^2 x}$

$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$

$= \sqrt{\frac{16}{25}}$ or $\frac{4}{5}$

$\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{4}{5}\right)^2}$

$= \sqrt{\frac{9}{25}}$ or $\frac{3}{5}$

$\cos(x - y) = \cos x \cos y + \sin x \sin y$

$= \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \frac{24}{25}$

28. $\cos x = \sqrt{1 - \sin^2 x}$

$= \sqrt{1 - \left(\frac{8}{17}\right)^2}$

$= \sqrt{\frac{225}{289}}$ or $\frac{15}{17}$

$\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$

$= \sqrt{\frac{16}{25}}$ or $\frac{4}{5}$

$\tan x = \frac{\sin x}{\cos x}$

$= \frac{\frac{8}{17}}{\frac{15}{17}}$

$= \frac{8}{15}$

$\tan y = \frac{\sin y}{\cos y}$

$= \frac{\frac{4}{5}}{\frac{3}{5}}$

$= \frac{4}{3}$

$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$= \frac{\frac{8}{15} - \frac{4}{3}}{1 + \frac{8}{15}\cdot\frac{4}{3}}$

$= \frac{-\frac{12}{15}}{\frac{77}{45}}$

$= -\frac{36}{77}$

29. $\sec x = \sqrt{\tan^2 x + 1}$ $\quad\cos y = \sqrt{1 - \sin^2 y}$

$= \sqrt{\left(\frac{5}{3}\right)^2 + 1}$ $\quad = \sqrt{1 - \left(\frac{1}{3}\right)^2}$

$= \sqrt{\frac{34}{9}}$ or $\frac{\sqrt{34}}{3}$ $\quad = \sqrt{\frac{8}{9}}$ or $\frac{2\sqrt{2}}{3}$

$\cos x = \frac{3}{\sqrt{34}}$ or $\frac{3\sqrt{34}}{34}$

$\tan x = \frac{\sin x}{\cos x}$

$\frac{5}{3} = \frac{\sin x}{\frac{3\sqrt{34}}{34}}$

$\sin x = \frac{5\sqrt{34}}{34}$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$= \left(\frac{3\sqrt{34}}{34}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{5\sqrt{34}}{34}\right)\left(\frac{1}{3}\right)$

$= \frac{6\sqrt{68}}{102} - \frac{5\sqrt{34}}{102}$

$= \frac{12\sqrt{17} - 5\sqrt{34}}{102}$

30. $\tan x = \frac{1}{\cot x}$ $\quad\cos y = \frac{1}{\sec y}$

$= \frac{1}{\frac{6}{5}}$ $\quad = \frac{1}{\frac{3}{2}}$

$= \frac{5}{6}$ $\quad = \frac{2}{3}$

$\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{2}{3}\right)^2}$

$= \sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$

$\tan y = \frac{\sin y}{\cos y}$

$= \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}$ or $\frac{\sqrt{5}}{2}$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$= \frac{\frac{5}{6} + \frac{\sqrt{5}}{2}}{1 - \left(\frac{5}{6}\right)\left(\frac{\sqrt{5}}{2}\right)}$

$= \frac{\frac{10 + 6\sqrt{5}}{12}}{\frac{12 - 5\sqrt{5}}{12}}$

$= \frac{10 + 6\sqrt{5}}{12 - 5\sqrt{5}}$

$= \frac{270 + 122\sqrt{5}}{19}$

31. $\sin x = \frac{1}{\csc x}$ $\quad\sec y = \sqrt{\tan^2 y + 1}$

$= \frac{1}{\frac{5}{3}}$ $\quad = \sqrt{\left(\frac{12}{5}\right)^2 + 1}$

$= \frac{3}{5}$ $\quad = \sqrt{\frac{169}{25}}$ or $\frac{13}{5}$

$\cos x = \sqrt{1 - \sin^2 x}$ $\quad\cos y = \frac{1}{\sec y}$

$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$ $\quad = \frac{1}{\frac{13}{5}}$

$= \sqrt{\frac{16}{25}}$ or $\frac{4}{5}$ $\quad = \frac{5}{13}$

$\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{5}{13}\right)^2}$

$= \sqrt{\frac{144}{169}}$ or $\frac{12}{13}$

$\cos(x - y) = \cos x \cos y + \sin x \sin y$

$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{56}{65}$

$\sec(x - y) = \frac{1}{\cos(x - y)}$

$= \frac{1}{\frac{56}{55}}$

$= \frac{65}{56}$

32. $\cos\alpha = \sqrt{1 - \sin^2\alpha}$ $\quad\sin\beta = \sqrt{1 - \cos^2\beta}$

$= \sqrt{1 - \left(\frac{1}{5}\right)^2}$ $\quad = \sqrt{1 - \left(\frac{2}{7}\right)^2}$

$= \sqrt{\frac{24}{25}}$ or $\frac{2\sqrt{6}}{5}$ $\quad = \sqrt{\frac{45}{49}}$ or $\frac{3\sqrt{5}}{7}$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{1}{5}\right)\left(\frac{2}{7}\right) - \left(\frac{2\sqrt{6}}{5}\right)\left(\frac{3\sqrt{5}}{7}\right)$

$= \frac{2 - 6\sqrt{30}}{35}$

33. $\sin x = \sqrt{1 - \cos^2 x}$ $\quad\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{1}{3}\right)^2}$ $\quad = \sqrt{1 - \left(\frac{3}{4}\right)^2}$

$= \sqrt{\frac{8}{9}}$ or $\frac{2\sqrt{2}}{3}$ $\quad = \sqrt{\frac{7}{16}}$ or $\frac{\sqrt{7}}{4}$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$= \left(\frac{1}{3}\right)\left(\frac{3}{4}\right) - \left(\frac{2\sqrt{2}}{3}\right)\left(\frac{\sqrt{7}}{4}\right)$

$= \frac{3 - 2\sqrt{14}}{12}$

34. $\cos\left(\frac{\pi}{2} + x\right) \stackrel{?}{=} -\sin x$

$\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x \stackrel{?}{=} -\sin x$

$0 \cdot \cos x - 1 \cdot \sin x \stackrel{?}{=} -\sin x$

$-\sin x = -\sin x$

35. $\cos(60° + A) \stackrel{?}{=} \sin(30° - A)$

$\cos 60° \cos A - \sin 60° \sin A \stackrel{?}{=} \sin 30° \cos A - \cos 30° \sin A$

$\frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A = \frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A$

36. $\sin(A + \pi) \stackrel{?}{=} -\sin A$

$\sin A \cos\pi + \cos A \sin\pi \stackrel{?}{=} -\sin A$

$(\sin A)(-1) + (\cos A)(0) \stackrel{?}{=} -\sin A$

$-\sin A = -\sin A$

37. $\cos(180° + x) \stackrel{?}{=} -\cos x$

$\cos 180° \cos x - \sin 180° \sin x \stackrel{?}{=} -\cos x$

$-1 \cdot \cos x - 0 \cdot \sin x \stackrel{?}{=} -\cos x$

$-\cos x = -\cos x$

38. $\tan(x + 45°) \stackrel{?}{=} \frac{1 + \tan x}{1 - \tan x}$

$\frac{\tan x + \tan 45°}{1 - \tan x \tan 45°} \stackrel{?}{=} \frac{1 + \tan x}{1 - \tan x}$

$\frac{\tan x + 1}{1 - (\tan x)(1)} \stackrel{?}{=} \frac{1 + \tan x}{1 - \tan x}$

$\frac{1 + \tan x}{1 - \tan x} = \frac{1 + \tan x}{1 - \tan x}$

39. $\sin(A + B) \stackrel{?}{=} \frac{\tan A + \tan B}{\sec A \sec B}$

$\sin(A + B) \stackrel{?}{=} \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}$

$\sin(A + B) \stackrel{?}{=} \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}} \cdot \frac{\cos A \cos B}{\cos A \cos B}$

$\sin(A + B) \stackrel{?}{=} \frac{\sin A \cos B + \cos A \sin B}{1}$

$\sin(A + B) = \sin(A + B)$

40. $\cos(A + B) \stackrel{?}{=} \frac{1 - \tan A \tan B}{\sec A \sec B}$

$\cos(A + B) \stackrel{?}{=} \frac{1 - \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}$

$\cos(A + B) \stackrel{?}{=} \frac{1 - \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}} \cdot \frac{\cos A \cos B}{\cos A \cos B}$

$\cos(A + B) \stackrel{?}{=} \frac{\cos A \cos B - \sin A \sin B}{1}$

$\cos(A + B) = \cos(A + B)$

41. $\sec(A - B) \stackrel{?}{=} \frac{\sec A \sec B}{1 + \tan A \tan B}$

$\sec(A - B) \stackrel{?}{=} \frac{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}{1 + \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}}$

$\sec(A - B) \stackrel{?}{=} \frac{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}{1 + \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}} \cdot \frac{\cos A \cos B}{\cos A \cos B}$

$\sec(A - B) \stackrel{?}{=} \frac{1}{\cos A \cos B + \sin A \sin B}$

$\sec(A - B) \stackrel{?}{=} \frac{1}{\cos(A - B)}$

$\sec(A - B) = \sec(A - B)$

42. $\sin(x + y)\sin(x - y) \stackrel{?}{=} \sin^2 x - \sin^2 y$

$(\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$
$\stackrel{?}{=} \sin^2 x - \sin^2 y$

$(\sin x \cos y)^2 - (\cos x \sin y)^2 \stackrel{?}{=} \sin^2 x - \sin^2 y$

$\sin^2 x \cos^2 y - \cos^2 x \sin^2 y \stackrel{?}{=} \sin^2 x - \sin^2 y$

$\sin^2 x \cos^2 y + \sin^2 x \sin^2 y - \sin^2 x \sin^2 y$
$- \cos^2 x \sin^2 y \stackrel{?}{=} \sin^2 x - \sin^2 y$

$\sin^2 x(\cos^2 y + \sin^2 y) - \sin^2 y(\sin^2 x + \cos^2 x)$
$\stackrel{?}{=} \sin^2 x - \sin^2 y$

$(\sin^2 x)(1) - (\sin^2 y)(1) \stackrel{?}{=} \sin^2 x - \sin^2 y$

$\sin^2 x - \sin^2 y = \sin^2 x - \sin^2 y$

43. $V_L = I_0 \omega L \cos\left(\omega t + \frac{\pi}{2}\right)$

$V_L = I_0 \omega L\left(\cos \omega t \cos \frac{\pi}{2} - \sin \omega t \sin \frac{\pi}{2}\right)$

$V_L = I_0 \omega L(\cos \omega t \cdot 0 - \sin \omega t \cdot 1)$

$V_L = I_0 \omega L(-\sin \omega t)$

$V_L = -I_0 \omega L \sin \omega t$

44. $n = \frac{\sin\left[\frac{1}{2}(\alpha + \beta)\right]}{\sin \frac{\beta}{2}}$

$n = \frac{\sin\left[\frac{1}{2}(\alpha + 60°)\right]}{\sin \frac{60°}{2}}$

$n = \frac{\sin\left(\frac{\alpha}{2} + 30°\right)}{\sin 30°}$

$n = \frac{\sin \frac{\alpha}{2} \cos 30° + \cos \frac{\alpha}{2} \sin 30°}{\frac{1}{2}}$

$n = 2\left[\left(\sin \frac{\alpha}{2}\right) \cdot \frac{\sqrt{3}}{2} + \left(\cos \frac{\alpha}{2}\right) \cdot \frac{1}{2}\right]$

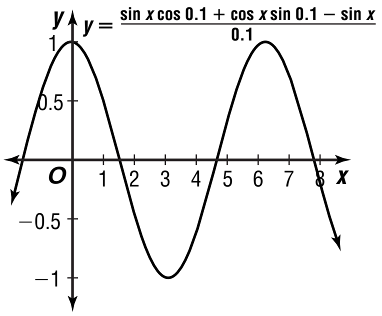
$n = \sqrt{3} \sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}$

45. The given expression is the expanded form of the sine of the difference of $\frac{\pi}{3} - A$ and $\frac{\pi}{3} + A$. We have

$\sin\left[\left(\frac{\pi}{3} - A\right) - \left(\frac{\pi}{3} + A\right)\right] = \sin(-2A)$
$= -\sin 2A$

46a. $\frac{f(x + h) - f(x)}{h} = \frac{\sin(x + h) - \sin x}{h}$
$= \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$

46b.



46c. $\cos x$

47. $\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$

$\tan(\alpha + \beta) = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}$

$\tan(\alpha + \beta) = \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$

$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

Replace β with $-\beta$ to find $\tan(\alpha - \beta)$.

$\tan(\alpha + (-\beta)) = \frac{\tan \alpha + \tan(-\beta)}{1 - \tan \alpha \tan(-\beta)}$

$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

48a. Answers will vary.

48b. $\tan A + \tan B + \tan C \stackrel{?}{=} \tan A \tan B \tan C$

$\tan A + \tan B + \tan(180° - (A + B))$

$\stackrel{?}{=} \tan A \tan B \tan(180° - (A + B))$

$\tan A + \tan B + \frac{\tan 180° - \tan(A + B)}{1 + \tan 180° \tan(A + B)}$

$\stackrel{?}{=} \tan A \tan B \frac{\tan 180° - \tan(A + B)}{1 + \tan 180° \tan(A + B)}$

$\tan A + \tan B + \frac{0 - \tan(A + B)}{1 + 0 \cdot \tan(A + B)}$

$\stackrel{?}{=} \tan A \tan B \frac{0 - \tan(A + B)}{1 + 0 \cdot \tan(A + B)}$

$\tan A + \tan B - \tan(A + B)$

$\stackrel{?}{=} -\tan A \tan B \tan(A + B)$

$(\tan A + \tan B) \cdot \frac{1 - \tan A \tan B}{1 - \tan A \tan B} - \tan(A + B)$

$\stackrel{?}{=} -\tan A \tan B (A + B)$

$\tan(A + B)(1 - \tan A \tan B) - \tan(A + B)$

$\stackrel{?}{=} -\tan A \tan B (A + B)$

$(1 - \tan A \tan B - 1) \tan(A + B)$

$\stackrel{?}{=} -\tan A \tan B (A + B)$

$-\tan A \tan B \tan(A + B) = -\tan A \tan B (A + B)$

49. $\sec^2 x \stackrel{?}{=} \frac{1 - \cos^2 x}{1 - \sin^2 x} + \csc^2 x - \cot^2 x$

$\sec^2 x \stackrel{?}{=} \frac{1 - \cos^2 x}{\cos^2 x} + 1 + \cot^2 x - \cot^2 x$

$\sec^2 x \stackrel{?}{=} \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} + 1$

$\sec^2 x \stackrel{?}{=} \sec^2 x - 1 + 1$

$\sec^2 x = \sec^2 x$

50. $\sin^2 \theta + \cos^2 \theta = 1$

$\left(-\frac{1}{8}\right)^2 + \cos^2 \theta = 1$

$\cos^2 \theta = \frac{63}{64}$

$\cos \theta = \pm\frac{3\sqrt{7}}{8}$

Quadrant III, so $-\frac{3\sqrt{7}}{8}$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

$= \frac{-\frac{1}{8}}{-\frac{3\sqrt{7}}{8}}$

$= \frac{1}{3\sqrt{7}}$

$= \frac{\sqrt{7}}{21}$

51. $\text{Arctan}\sqrt{3} = \frac{\pi}{3}$

$\sin(\text{Arctan}\sqrt{3}) = \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

52. πk, where k is an integer

53. $A = \frac{86 - 50}{2}$ $\qquad \frac{2\pi}{4} = \frac{\pi}{2}$ $\qquad h = \frac{86 + 50}{2}$

$A = 18 = 68$ $\qquad\qquad = 68$

$y = 18 \sin\left(\frac{\pi}{2}t + c\right) + 68$

$50 = 18 \sin\left(\frac{\pi}{2} \cdot 1 + c\right) + 68$

$-18 = 18 \sin\left(\frac{\pi}{2} + c\right)$

$-1 = \sin\left(\frac{\pi}{2} + c\right)$

$\sin^{-1}(-1) = \frac{\pi}{2} + c$

$\frac{3\pi}{2} = \frac{\pi}{2} + c$

$\pi = c$

$y = 18\sin\left(\frac{\pi}{2}t - \pi\right) + 68$

54. $|8| = 8$; $\frac{360}{1} = 360$; $\frac{30°}{1} = 30°$

55. $\sin(-540°) = \sin(-360° - 180°)$

$= 0$

56. $s = r\theta$ $\qquad A = \frac{1}{2}r^2\theta$

$18 = r(2.9)$ $\qquad A \approx \frac{1}{2}(6.2)^2(2.9)$

$6.2 \approx r$; 6.2 ft $\qquad A \approx 55.7\text{ ft}^2$

57. $c^2 = 70^2 + 130^2 - 2(70)(130)\cos 130°$

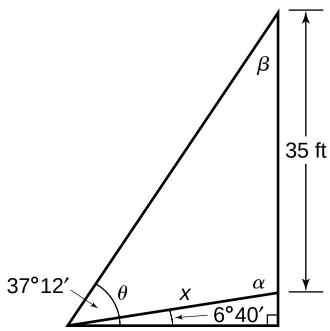
$c^2 \approx 33498.7345$

$c \approx 183$ miles

58. $120° \geq 90°$, consider Case 2.

$4 \leq 12$, 0 solutions

59.



$\theta = 37°\,12' - 6°\,40'$ or $30°\,32'$

$\alpha = 90° + 6°\,40'$ or $96°\,40'$

$\beta = 180° - (30°\,32' + 96°\,40')$ or $52°\,48'$

$\frac{35}{\sin 30°\,32'} = \frac{x}{\sin 52°\,48'}$

$x = \frac{35 \sin 52°\,48'}{\sin 30°\,32'}$

$x \approx 54.87$ ft

60. $4x^3 + 3x^2 - x = 0$

$x(4x^2 + 3x - 1) = 0$

$x(4x - 1)(x + 1) = 0$

$x = 0$ or $4x - 1 = 0$ or $x + 1 = 0$

$x = \frac{1}{4}$ $\qquad x = -1$

61. Case 1

$|x + 1| > 4$

$-(x + 1) > 4$

$-x - 1 > 4$

$-x > 5$

$x < -5$

Case 2

$|x + 1| > 4$

$x + 1 > 4$

$x > 3$

$\{x \mid x < -5 \text{ or } x > 3\}$

62. $\begin{vmatrix} -1 & -2 \\ 3 & -6 \end{vmatrix} = -1(-6) - 3(-2)$

$= 6 + 6$ or 12

63. $f \circ g(4) = f(g(4))$

$= f(5(4) + 1)$

$= f(21)$

$= 3(21)^2 - 4$

$= 1319$

$g \circ f(4) = g(f(4))$

$= g(3(4)^2 - 4)$

$= g(44)$

$= 5(44) + 1$

$= 221$

64. $(-8)^{62} \div 8^{62} = \frac{(-8)^{62}}{8^{62}} = \left(\frac{-8}{8}\right)^{62} = (-1)^{62} = 1$

The correct choice is A.

Page 445 Mid-Chapter Quiz

1. $\csc\theta = \frac{1}{\sin\theta}$
$= \dfrac{1}{\frac{2}{7}}$
$= \frac{7}{2}$

$1 + \cot^2\theta = \csc^2\theta$
$1 + \cot^2\theta = \left(\frac{7}{2}\right)^2$
$1 + \cot^2\theta = \frac{49}{4}$
$\cot^2\theta = \frac{45}{4}$
$\cot\theta = \pm\frac{3\sqrt{5}}{2}$

Quadrant 1, so $\frac{3\sqrt{5}}{2}$

2. $\tan^2\theta + 1 = \sec^2\theta$
$\left(-\frac{4}{3}\right)^2 + 1 = \sec^2\theta$
$\frac{16}{9} + 1 = \sec^2\theta$
$\frac{25}{9} = \sec^2\theta$
$\pm\frac{5}{3} = \sec\theta$

$\cos\theta = \frac{1}{\sec\theta}$
$= \dfrac{1}{-\frac{5}{3}}$
$= -\frac{3}{5}$

Quadrant II, so $-\frac{5}{3}$

3. $\frac{19\pi}{4} = 5\pi - \frac{\pi}{4}$
$\cos\frac{19\pi}{4} = \cos\left(5\pi - \frac{\pi}{4}\right)$
$= -\cos\frac{\pi}{4}$

4. $\frac{1}{1 + \tan^2 x} + \frac{1}{1 + \cot^2 x} \stackrel{?}{=} 1$
$\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} \stackrel{?}{=} 1$
$\cos^2 x + \sin^2 x \stackrel{?}{=} 1$
$1 = 1$

5. $\frac{\csc^2\theta + \sec^2\theta}{\sec^2\theta} \stackrel{?}{=} \csc^2\theta$
$\frac{\csc^2\theta}{\sec^2\theta} + \frac{\sec^2\theta}{\sec^2\theta} \stackrel{?}{=} \csc^2\theta$
$\dfrac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta}} + 1 \stackrel{?}{=} \csc^2\theta$
$\frac{\cos^2\theta}{\sin^2\theta} + 1 \stackrel{?}{=} \csc^2\theta$
$\cot^2\theta + 1 \stackrel{?}{=} \csc^2\theta$
$\csc^2\theta = \csc^2\theta$

6. $\cot x \sec x \sin x \stackrel{?}{=} 2 - \tan x \cos x \csc x$
$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} \cdot \sin x \stackrel{?}{=} 2 - \frac{\sin x}{\cos x} \cdot \cos x \cdot \frac{1}{\sin x}$
$1 \stackrel{?}{=} 2 - 1$
$1 = 1$

7. $\tan(\alpha - \beta) \stackrel{?}{=} \frac{1 - \cot\alpha\tan\beta}{\cot\alpha + \tan\beta}$
$\tan(\alpha - \beta) \stackrel{?}{=} \dfrac{1 - \frac{1}{\tan\alpha}\cdot\tan\beta}{\frac{1}{\tan\alpha} + \tan\beta}$
$\tan(\alpha - \beta) \stackrel{?}{=} \dfrac{1 - \frac{1}{\tan\alpha}\cdot\tan\beta}{\frac{1}{\tan\alpha} + \tan\beta} \cdot \frac{\tan\alpha}{\tan\alpha}$
$\tan(\alpha - \beta) \stackrel{?}{=} \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$
$\tan(\alpha - \beta) = \tan(\alpha - \beta)$

8. $\cos 75° = \cos(30° + 45°)$
$= \cos 30°\cos 45° - \sin 30°\sin 45°$
$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

9. $\cos x = \sqrt{1 - \sin^2 x}$
$= \sqrt{1 - \left(\frac{2}{3}\right)^2}$
$= \sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$

$\cos y = \sqrt{1 - \sin^2 y}$
$= \sqrt{1 - \left(\frac{3}{4}\right)^2}$
$= \sqrt{\frac{7}{16}}$ or $\frac{\sqrt{7}}{4}$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$
$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{\sqrt{7}}{4}\right) - \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$
$= \frac{\sqrt{35} - 6}{12}$

10. $\tan x = \frac{5}{4}$

$\tan y = \sqrt{\sec^2 y - 1}$
$= \sqrt{2^2 - 1}$
$= \sqrt{3}$

$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$
$= \dfrac{\frac{5}{4} - \sqrt{3}}{1 + \left(\frac{5}{4}\right)\left(\sqrt{3}\right)}$
$= \dfrac{\frac{5 - 4\sqrt{3}}{4}}{\frac{4 + 5\sqrt{3}}{4}}$
$= \frac{5 - 4\sqrt{3}}{4 + 5\sqrt{3}}$
$= \frac{80 - 41\sqrt{3}}{-59}$ or $\frac{-80 + 41\sqrt{3}}{59}$

7-3B Reduction Identities

Page 447

1. −sin, −cos, sin
2. −cot, tan, −cot
3. −tan, cot, −tan
4. −csc, −sec, csc
5. sec, −csc, −sec

6a. (1) −cos, −sin, cos
(2) sin, −cos, −sin
(3) −cot, tan, −cot
(4) −tan, cot, −tan
(5) csc, −sec, −csc
(6) −sec, −csc, sec

6b. Sample answer: If a row for sin α were placed above Exercises 1-5, the entries for Exercise 6a could be obtained by interchanging the first and third columns and leaving the middle column alone.

7a. (1) cos, sin, −cos
(2) sin, −cos, −sin
(3) cot, −tan, cot
(4) tan, −cot, tan
(5) csc, −sec, −csc
(6) sec, csc, −sec

7b. Sample answer: The entries in the rows for cos α and sec α are unchanged. All other entries are multiplied by −1.

8a. Sample answer: They can be used to reduce trigonometric functions of large positive or negative angles to those of angles in the first quadrant.

8b. Sample answer: sum or difference identities

7-4 Double-Angle and Half-Angle Identities

Page 453 Check for Understanding

1. If you are only given the value of $\cos\theta$, then $\cos 2\theta = 2\cos^2\theta - 1$ is the best identity to use. If you are only given the value of $\sin\theta$, then $\cos 2\theta = 1 - 2\sin^2\theta$ is the best identity to use. If you are given the values of both $\cos\theta$ and $\sin\theta$, then $\cos 2\theta = \cos^2\theta - \sin^2\theta$ is just as good as the other two.

2. $$\cos 2\pi = 1 - 2\sin^2\theta$$
$$\cos 2\theta - 1 = -2\sin^2\theta$$
$$\frac{\cos 2\theta - 1}{-2} = \sin^2\theta$$
$$\frac{1 - \cos 2\theta}{2} = \sin^2\theta$$
$$\pm\sqrt{\frac{1 - \cos 2\theta}{2}} = \sin\theta$$
Letting $\theta = \frac{\alpha}{2}$ yields $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos 2\left(\frac{\alpha}{2}\right)}{2}}$, or $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$.

3a. III or IV **3b.** I or II **3c.** I, II, III or IV

4. $$\sin 2\theta \stackrel{?}{=} 2\sin\theta$$
$$\sin 2\left(\frac{\pi}{2}\right) \stackrel{?}{=} 2\sin\frac{\pi}{2}$$
$$\sin\pi \stackrel{?}{=} 2\sin\frac{\pi}{2}$$
$$0 \stackrel{?}{=} 2(1)$$
$$0 \neq 2$$
Sample answer: $\theta = \frac{\pi}{2}$

5. Both answers are correct. She obtained two different representations of the same number. One way to verify this is to evaluate each expression with a calculator. To verify it algebraically, square each answer and then simplify. The same result is obtained in each case. Since each of the original answers is positive, and they have the same square, the original answers are the same number.

6. $$\sin\frac{\pi}{8} = \sin\frac{\frac{\pi}{4}}{2}$$
$$= \sqrt{\frac{1 - \cos\frac{\pi}{4}}{2}} \quad \text{(Quadrant I)}$$
$$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$
$$= \frac{\sqrt{2 - \sqrt{2}}}{2}$$

7. $$\tan 165° = \tan\frac{330°}{2}$$
$$= \sqrt{\frac{1 - \cos 330°}{1 + \cos 330°}} \quad \text{(Quadrant II)}$$
$$= -\sqrt{\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}}$$
$$= -(2 - \sqrt{3})$$
$$= \sqrt{3} - 2$$

8. $$\sin^2\theta + \cos^2\theta = 1$$
$$\left(\frac{2}{5}\right)^2 + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{21}{25}$$
$$\cos\theta = \frac{\sqrt{21}}{5}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{5}}{\frac{\sqrt{21}}{5}} = \frac{2}{\sqrt{21}} \text{ or } \frac{2\sqrt{21}}{21} \quad \text{(Quadrant I)}$$
$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{2}{5}\right)\left(\frac{\sqrt{21}}{5}\right) = \frac{4\sqrt{21}}{25}$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{\sqrt{21}}{5}\right)^2 - \left(\frac{2}{5}\right)^2 = \frac{17}{25}$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2\left(\frac{2\sqrt{21}}{21}\right)}{1 - \left(\frac{2\sqrt{21}}{21}\right)^2} = \frac{\frac{4\sqrt{21}}{21}}{\frac{17}{21}} \text{ or } \frac{4\sqrt{21}}{17}$$

9. $$\tan^2\theta + 1 = \sec^2\theta$$
$$\left(\frac{4}{3}\right)^2 + 1 = \sec^2\theta$$
$$\frac{25}{9} = \sec^2\theta$$
$$-\frac{5}{3} = \sec\theta \quad \text{(Quadrant III)}$$
$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{-5}{3}} \text{ or } -\frac{3}{5}$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(-\frac{3}{5}\right)^2 = 1$$
$$\sin^2\theta = \frac{16}{25}$$
$$\sin\theta = -\frac{4}{5} \quad \text{(Quadrant III)}$$
$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \frac{24}{25}$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 = -\frac{7}{25}$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{-\frac{7}{9}} \text{ or } -\frac{24}{7}$$

10. $$\tan 2\theta \stackrel{?}{=} \frac{2}{\cot\theta - \tan\theta}$$
$$\tan 2\theta \stackrel{?}{=} \frac{2}{\cot\theta - \tan\theta} \cdot \frac{\tan\theta}{\tan\theta}$$
$$\tan 2\theta \stackrel{?}{=} \frac{2\tan\theta}{\cot\theta\tan\theta - \tan^2\theta}$$
$$\tan 2\theta \stackrel{?}{=} \frac{2\tan\theta}{1 - \tan^2\theta}$$
$$\tan 2\theta = \tan 2\theta$$

11. $1 + \frac{1}{2}\sin 2A \stackrel{?}{=} \frac{\sec A + \sin A}{\sec A}$

$1 + \frac{1}{2}\sin 2A \stackrel{?}{=} \dfrac{\frac{1}{\cos A} + \sin A}{\frac{1}{\cos A}}$

$1 + \frac{1}{2}\sin 2A \stackrel{?}{=} \dfrac{\frac{1}{\cos A} + \sin A}{\frac{1}{\cos A}} \cdot \frac{\cos A}{\cos A}$

$1 + \frac{1}{2}\sin 2A \stackrel{?}{=} 1 + \sin A\cos A$

$1 + \frac{1}{2}\sin 2A \stackrel{?}{=} 1 + \frac{1}{2}\cdot 2\sin A\cos A$

$1 + \frac{1}{2}\sin 2A = 1 + \frac{1}{2}\sin 2A$

12. $\sin\frac{x}{2}\cos\frac{x}{2} \stackrel{?}{=} \frac{\sin x}{2}$

$\dfrac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2} \stackrel{?}{=} \frac{\sin x}{2}$

$\dfrac{\sin 2\left(\frac{x}{2}\right)}{2} \stackrel{?}{=} \frac{\sin x}{2}$

$\frac{\sin x}{2} = \frac{\sin x}{2}$

13. $\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta + 1 = 2\cos^2\theta$

$\frac{1}{2}\cos 2\theta + \frac{1}{2} = \cos^2\theta$

$P = I_0{}^2 R\sin^2\omega t$

$P = I_0{}^2 R(1 - \cos^2\omega t)$

$P = I_0{}^2 R\left(1 - \left(\frac{1}{2}\cos 2\omega t + \frac{1}{2}\right)\right)$

$P = I_0{}^2 R\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right)$

$P = \frac{1}{2}I_0{}^2 R - \frac{1}{2}I_0{}^2 R\cos 2\omega t$

Pages 454–455 Exercises

14. $\cos 15° = \cos\frac{30°}{2}$

$= \sqrt{\frac{1 + \cos 30°}{2}}$ (Quadrant I)

$= \sqrt{\dfrac{1 + \frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{2 + \sqrt{3}}{2}}$

15. $\sin 75° = \sin\frac{150°}{2}$

$= \sqrt{\frac{1 - \cos 150°}{2}}$ (Quadrant I)

$= \sqrt{\dfrac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}}$

$= \sqrt{\frac{2 + \sqrt{3}}{2}}$

16. $\tan\frac{5\pi}{12} = \tan\dfrac{\frac{5\pi}{6}}{2}$

$= \sqrt{\dfrac{1 - \cos\frac{5\pi}{6}}{1 + \cos\frac{5\pi}{6}}}$ (Quadrant I)

$= \sqrt{\dfrac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{1 + \left(-\frac{\sqrt{3}}{2}\right)}}$

$= \sqrt{\dfrac{\frac{2 + \sqrt{3}}{2}}{\frac{2 - \sqrt{3}}{2}}}$

$= \sqrt{\frac{(2 + \sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}}$

$= \sqrt{\frac{(2 + \sqrt{3})^2}{4 - 3}}$

$= 2 + \sqrt{3}$

17. $\sin\frac{3\pi}{8} = \sin\dfrac{\frac{3\pi}{4}}{2}$

$= \sqrt{\dfrac{1 - \cos\frac{3\pi}{4}}{2}}$ (Quadrant I)

$= \sqrt{\dfrac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2}}$

$= \sqrt{\frac{\sqrt{2} + \sqrt{2}}{2}}$

18. $\cos\frac{7\pi}{12} = \cos\dfrac{\frac{7\pi}{6}}{2}$

$= -\sqrt{\dfrac{1 + \cos\frac{7\pi}{6}}{2}}$ (Quadrant II)

$= -\sqrt{\dfrac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2}}$

$= -\frac{\sqrt{2 - \sqrt{3}}}{2}$

19. $\tan 22.5° = \tan\frac{45°}{2}$

$= \sqrt{\frac{1 - \cos 45°}{1 + \cos 45°}}$ (Quadrant I)

$= \sqrt{\dfrac{1 - \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}}}$

$= \sqrt{\dfrac{\frac{2 - \sqrt{2}}{2}}{\frac{2 + \sqrt{2}}{2}}}$

$= \sqrt{\frac{(2 - \sqrt{2})(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})}}$

$= \sqrt{\frac{(2 - \sqrt{2})^2}{4 - 2}}$

$= \sqrt{\frac{2 - \sqrt{2}}{\sqrt{2}}}$

$= \frac{2\sqrt{2} - 2}{2}$

$= \sqrt{2} - 1$

20. $\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{1-\frac{1}{4}}{1+\frac{1}{4}}}$

$= \sqrt{\frac{\frac{3}{4}}{\frac{5}{4}}}$

$= \sqrt{\frac{3}{5}}$ or $\frac{\sqrt{15}}{5}$

21. $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{4}{5}\right)^2 = 1$

$\sin^2\theta = \frac{9}{25}$

$\sin\theta = \frac{3}{5}$

(Quadrant I)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{3}{5}}{\frac{4}{5}}$

$= \frac{3}{4}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{25}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{7}{25}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$

$= \frac{\frac{3}{2}}{\frac{7}{16}}$ or $\frac{24}{7}$

22. $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{8}{9}$

$\cos\theta = \frac{2\sqrt{2}}{3}$

(Quadrant I)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}$

$= \frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$

$= \frac{4\sqrt{2}}{9}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2$

$= \frac{7}{9}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{\sqrt{2}}{4}\right)}{1-\left(\frac{\sqrt{2}}{4}\right)^2}$

$= \frac{\frac{\sqrt{2}}{2}}{\frac{14}{16}}$ or $\frac{4\sqrt{2}}{7}$

23. $\tan^2\theta + 1 = \sec^2\theta$

$(-2)^2 + 1 = \sec^2\theta$

$5 = \sec^2\theta$

$-\sqrt{5} = \sec\theta$ (Quadrant II)

$\cos\theta = \sec\theta$

$= \frac{1}{-\sqrt{5}}$ or $-\frac{\sqrt{5}}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{\sqrt{5}}{5}\right)^2 = 1$

$\sin^2\theta = \frac{20}{25}$

$\sin\theta = \frac{2\sqrt{5}}{5}$ (Quadrant II)

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{2\sqrt{5}}{5}\right)\left(-\frac{\sqrt{5}}{5}\right)$

$= -\frac{4}{5}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{\sqrt{5}}{5}\right)^2 - \left(\frac{2\sqrt{5}}{5}\right)^2$

$= -\frac{3}{5}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2(-2)}{1-(-2)^2}$

$= \frac{-4}{-3}$ or $\frac{4}{3}$

24. $\cos\theta = \frac{1}{\sec\theta}$

$= \frac{1}{-\frac{4}{3}}$

$= -\frac{3}{4}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{3}{4}\right)^2 = 1$

$\sin^2\theta = \frac{7}{16}$

$\sin\theta = \frac{\sqrt{7}}{4}$ (Quadrant II)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{\sqrt{7}}{4}}{-\frac{3}{4}}$

$= -\frac{\sqrt{7}}{3}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{7}}{4}\right)\left(-\frac{3}{4}\right)$

$= -\frac{3\sqrt{7}}{8}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{3}{4}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2$

$= \frac{2}{16}$ or $\frac{1}{8}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(-\frac{\sqrt{7}}{3}\right)}{1-\left(-\frac{\sqrt{7}}{3}\right)^2}$

$= -\frac{\frac{2\sqrt{7}}{3}}{\frac{2}{9}}$ or $-3\sqrt{7}$

25. $1 + \cot^2\theta = \csc^2\theta$

$1 + \left(\frac{3}{2}\right)^2 = \csc^2\theta$

$\frac{13}{4} = \csc^2\theta$

$-\frac{\sqrt{13}}{2} = \csc\theta$

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

(Quadrant III)

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{\sqrt{13}}{2}} = -\frac{2}{\sqrt{13}}$ or $-\frac{2\sqrt{13}}{13}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{2\sqrt{13}}{13}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{117}{169}$

$\cos\theta = -\frac{3\sqrt{13}}{13}$

(Quadrant III)

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{2\sqrt{13}}{13}\right)\left(-\frac{3\sqrt{13}}{13}\right) = \frac{12}{13}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{3\sqrt{13}}{13}\right)^2 - \left(-\frac{2\sqrt{13}}{13}\right)^2 = \frac{5}{13}$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2\left(\frac{2}{3}\right)}{1 - \left(\frac{2}{3}\right)^2} = \frac{\frac{4}{3}}{\frac{5}{9}}$ or $\frac{12}{5}$

26. $\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{5}{2}} = -\frac{2}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{2}{5}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{21}{25}$

$\cos\theta = \frac{\sqrt{21}}{5}$

(Quadrant IV)

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{2}{5}}{\frac{\sqrt{21}}{5}} = -\frac{2}{\sqrt{21}}$ or $-\frac{2\sqrt{21}}{21}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{2}{5}\right)\left(\frac{\sqrt{21}}{5}\right) = -\frac{4\sqrt{21}}{25}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{\sqrt{21}}{5}\right)^2 - \left(-\frac{2}{5}\right)^2 = \frac{17}{25}$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2\left(-\frac{2\sqrt{21}}{21}\right)}{1 - \left(-\frac{2\sqrt{21}}{21}\right)^2} = \frac{-\frac{4\sqrt{21}}{21}}{\frac{17}{21}}$ or $-\frac{4\sqrt{21}}{17}$

27. $\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \left(-\frac{\sqrt{2}}{3}\right)^2 = 1$

$\sin^2\alpha = \frac{7}{9}$

$\sin\alpha = \frac{\sqrt{7}}{3}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{7}}{3}}{-\frac{\sqrt{2}}{3}} = -\frac{\sqrt{7}}{\sqrt{2}}$ or $-\frac{\sqrt{14}}{2}$

(Quadrant II)

$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{2\left(-\frac{\sqrt{14}}{2}\right)}{1 - \left(-\frac{\sqrt{14}}{2}\right)^2} = \frac{-\sqrt{14}}{-\frac{5}{2}}$ or $\frac{2\sqrt{14}}{5}$

28. $\csc 2\theta \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{\sin 2\theta} \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2\sin\theta\cos\theta} \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2}\cdot\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2}\csc\theta\sec\theta \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2}\sec\theta\csc\theta = \frac{1}{2}\sec\theta\csc\theta$

29. $\cos A - \sin A \stackrel{?}{=} \frac{\cos 2A}{\cos A + \sin A}$

$\cos A - \sin A \stackrel{?}{=} \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A}$

$\cos A - \sin A \stackrel{?}{=} \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A + \sin A}$

$\cos A - \sin A = \cos A - \sin A$

30. $(\sin\theta + \cos\theta)^2 - 1 \stackrel{?}{=} \sin 2\theta$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1 \stackrel{?}{=} \sin 2\theta$

$2\sin\theta\cos\theta + 1 - 1 \stackrel{?}{=} \sin 2\theta$

$2\sin\theta\cos\theta \stackrel{?}{=} \sin 2\theta$

$\sin 2\theta = \sin 2\theta$

31. $\cos x - 1 \stackrel{?}{=} \frac{\cos 2x - 1}{2(\cos x + 1)}$

$\cos x - 1 \stackrel{?}{=} \frac{2\cos^2 x - 1 - 1}{2(\cos x + 1)}$

$\cos x - 1 \stackrel{?}{=} \frac{2\cos^2 x - 2}{2(\cos x + 1)}$

$\cos x - 1 \stackrel{?}{=} \frac{2(\cos^2 x - 1)}{2(\cos x + 1)}$

$\cos x - 1 \stackrel{?}{=} \frac{2(\cos x - 1)(\cos x + 1)}{2(\cos x + 1)}$

$\cos x - 1 = \cos x - 1$

32. $\sec 2\theta \stackrel{?}{=} \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$

$\sec 2\theta \stackrel{?}{=} \frac{1}{\cos 2\theta}$

$\sec 2\theta = \sec 2\theta$

33. $\tan\frac{A}{2} \stackrel{?}{=} \frac{\sin A}{1 + \cos A}$

$$\tan \frac{A}{2} \stackrel{?}{=} \frac{\sin 2\left(\frac{A}{2}\right)}{1 + \cos 2\left(\frac{A}{2}\right)}$$

$$\tan \frac{A}{2} \stackrel{?}{=} \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{1 + 2\cos^2 \frac{A}{2} - 1}$$

$$\tan \frac{A}{2} \stackrel{?}{=} \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{2\cos^2 \frac{A}{2}}$$

$$\tan \frac{A}{2} \stackrel{?}{=} \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}}$$

$$\tan \frac{A}{2} = \tan \frac{A}{2}$$

34.
$$\begin{aligned} \sin 3x &\stackrel{?}{=} 3 \sin x - 4 \sin^3 x \\ \sin(2x + x) &\stackrel{?}{=} 3 \sin x - 4 \sin^3 x \\ \sin 2x \cos x + \cos 2x \sin x &\stackrel{?}{=} 3 \sin x - 4 \sin^3 x \\ 2 \sin x \cos^2 x + (1 - 2\sin^2 x) \sin x &\stackrel{?}{=} 3 \sin x - 4 \sin^3 x \\ 2 \sin x(1 - \sin^2 x) + (1 - 2\sin^2 x) \sin x &\stackrel{?}{=} 3 \sin x - 4 \sin^3 x \\ 2 \sin x - 2\sin^3 x + \sin x - 2 \sin^3 x &\stackrel{?}{=} 3 \sin x - 4 \sin^3 x \\ 3 \sin x - 4 \sin^3 x &= 3 \sin x - 4 \sin^3 x \end{aligned}$$

35.
$$\begin{aligned} \cos 3x &\stackrel{?}{=} 4\cos^3 x - 3 \cos x \\ \cos(2x + x) &\stackrel{?}{=} 4\cos^3 x - 3 \cos x \\ (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x &\stackrel{?}{=} 4\cos^3 x - 3 \cos x \\ (2\cos^2 x - 1)\cos x - 2(1 - \cos^2 x)\cos x &\stackrel{?}{=} 4\cos^3 x - 3 \cos x \\ 2\cos^3 x - \cos x - 2 \cos x + 2\cos^3 x &\stackrel{?}{=} 4\cos^3 x - 3 \cos x \\ 4\cos^3 x - 3 \cos x &= 4\cos^3 x - 3 \cos x \end{aligned}$$

36.
$$\begin{aligned} \frac{\frac{v^2}{2g}\sin^2 2\theta}{\frac{v^2}{2g}\sin^2 \theta} &= \frac{\sin^2 2\theta}{\sin^2 \theta} \\ &= \frac{(2 \sin\theta \cos\theta)^2}{\sin^2\theta} \\ &= \frac{4\sin^2\theta\cos^2\theta}{\sin^2\theta} \\ &= 4\cos^2\theta \end{aligned}$$

37. $\angle PBD$ is an inscribed angle that subtends the same arc as the central angle $\angle POD$, so $m\angle PBD = \frac{1}{2}\theta$. By right triangle trigonometry, $\tan\frac{1}{2}\theta = \frac{PA}{BA} = \frac{PA}{1 + OA} = \frac{\sin\theta}{1 + \cos\theta}$.

38.
$$R = \frac{2v^2 \cos\theta \sin(\theta - \alpha)}{g\cos^2\alpha}$$

$$R = \frac{2v^2 \cos\theta \sin(\theta - 45°)}{g\cos^2 45°}$$

$$R = \frac{2v^2 \cos\theta(\sin\theta\cos 45° - \cos\theta \sin 45°)}{g\cos^2 45°}$$

$$R = \frac{2v^2\cos\theta\left((\sin\theta)\left(\frac{\sqrt{2}}{2}\right) - (\cos\theta)\left(\frac{\sqrt{2}}{2}\right)\right)}{g\left(\frac{\sqrt{2}}{2}\right)2}$$

$$R = \frac{2\frac{\sqrt{2}}{2}v^2\cos\theta(\sin\theta - \cos\theta)}{g \cdot \frac{1}{2}}$$

$$R = \frac{\sqrt{2}v^2\,(2\cos\theta\sin\theta - 2\cos^2\theta)}{g}$$

$$R = \frac{v^2\sqrt{2}}{g}(2\cos\theta\sin\theta - (2\cos^2\theta - 1) - 1)$$

$$R = \frac{v^2\sqrt{2}}{g}(\sin 2\theta - \cos 2\theta - 1)$$

39a.
$$\begin{aligned} \tan\left(45° + \frac{1}{2}\right) &= \frac{\tan 45° + \tan\frac{L}{2}}{1 - \tan 45° \tan\frac{L}{2}} \\ &= \frac{1 + \tan\frac{L}{2}}{1 - 1 \cdot \tan\frac{L}{2}} \\ &= \frac{1 \pm \sqrt{\frac{1 - \cos L}{1 + \cos L}}}{1 \mp \sqrt{\frac{1 - \cos L}{1 + \cos L}}} \end{aligned}$$

39b.
$$\begin{aligned} \frac{1 \pm \sqrt{\frac{1 - \cos L}{1 + \cos L}}}{1 \mp \sqrt{\frac{1 - \cos L}{1 + \cos L}}} &= \frac{1 + \sqrt{\frac{1 - \cos 60°}{1 + \cos 60°}}}{1 - \sqrt{\frac{1 - \cos 60°}{1 + \cos 60°}}} \\ &= \frac{1 + \sqrt{\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}}}}{1 - \sqrt{\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}}}} \\ &= \frac{1 + \sqrt{\frac{1}{3}}}{1 - \sqrt{\frac{1}{3}}} \\ &= \frac{\frac{3 + \sqrt{3}}{3}}{\frac{3 - \sqrt{3}}{3}} \\ &= \frac{12 + 6\sqrt{3}}{6} \text{ or } 2 + \sqrt{3} \end{aligned}$$

40.
$$\begin{aligned} \tan(\alpha + 30°) &= \tfrac{21}{7} \\ \tan(\alpha + 30°) &= 3 \\ \frac{\tan\alpha + \tan 30°}{1 - \tan\alpha\tan 30°} &= 3 \\ \tan\alpha + \frac{\sqrt{3}}{3} &= 3 - \sqrt{3}\tan\alpha \\ \tan\alpha + \sqrt{3}\tan\alpha &= 3 - \frac{\sqrt{3}}{3} \\ (1 + \sqrt{3})\tan\alpha &= 3 - \frac{\sqrt{3}}{3} \\ \tan\alpha &= \frac{3 - \frac{\sqrt{3}}{3}}{1 + \sqrt{3}} \\ \tan\alpha &= \frac{9 - \sqrt{3}}{3 + 3\sqrt{3}} \\ \tan\alpha &= \frac{-6 + 5\sqrt{3}}{3} \end{aligned}$$

41.
$$\begin{aligned} \cos\frac{\pi}{12} &= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4} \\ &= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4} \\ \sec\frac{\pi}{12} &= \frac{1}{\cos\frac{\pi}{12}} \\ &= \frac{1}{\frac{\sqrt{2} + \sqrt{6}}{4}} \\ &= \frac{4\sqrt{2} - 4\sqrt{6}}{-4} \text{ or } \sqrt{6} - \sqrt{2} \end{aligned}$$

42. Sample answer:
$$\begin{aligned} \sin(\sqrt{\pi})^2 + \cos(\sqrt{\pi})^2 &= \sin\pi + \cos\pi \\ &= 0 + (-1) \\ &= -1 \\ &\neq 1 \end{aligned}$$

43. $s = r\theta$

$17 = 10 \cdot \theta$

$\frac{17}{10} = \theta$

$\frac{17}{10} = \frac{17}{10} \cdot \frac{180°}{\pi}$

$\approx 97.4°$

44. Let x = the distance from A to the point beneath the mountain peak.

$\tan 21°10' = \frac{h}{570 + x}$

$h = (570 + x) \tan 21°10'$

$\tan 36°40' = \frac{h}{x}$

$h = x \tan 36°40'$

$(570 + x) \tan 21°10' = x \tan 36°40'$

$570 \tan 21°10' = x \tan 36°40' - x \tan 21°10'$

$570 \tan 21°10' = x(\tan 36°40' - \tan 21°10')$

$\frac{570 \tan 21°10'}{\tan 36°40' - \tan 21°10'} = x$

$617.7646751 \approx x$

$\tan 36°40' = \frac{h}{x}$

$\tan 36°40' \approx \frac{h}{617.8}$

$h \approx 460$ ft

45. $(x - (-3))(x - 0.5)(x - 6)(x - 2) = 0$

$(x + 3)(x - 0.5)(x - 6)(x - 2) = 0$

$(x^2 + 2.5x - 1.5)(x^2 - 8x + 12) = 0$

$x^4 - 5.5x^3 - 9.5x^2 + 42x - 18 = 0$

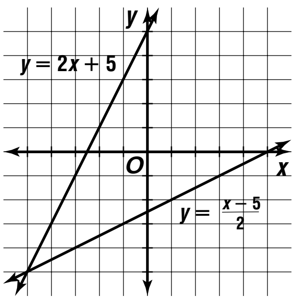
$2x^4 - 11x^3 - 19x^2 + 84x - 36 = 0$

46. $y = 2x + 5$

$x = 2y + 5$

$x - 5 = 2y$

$\frac{x-5}{2} = y$



47. $x + 2y = 11$

$x = 11 - 2y$

$3x - 5y = 11$

$3(11 - 2y) - 5y = 11$

$33 - 6y - 5y = 11$

$-11y = -22$

$y = 2$

$x + 2y = 11$

$x + 2(2) = 11$

$x = 7 \quad (7, 2)$

48. $ab = 3$

$b = \frac{3}{a}$

$(a - b)^2 = 64$

$a^2 - 2ab + b^2 = 64$

$a^2 - 2a\left(\frac{3}{a}\right) + \left(\frac{3}{a}\right)^2 = 64$

$a^2 - 6 + \left(\frac{3}{a}\right)^2 = 64$

$a^2 + \left(\frac{3}{a}\right)^2 = 70$

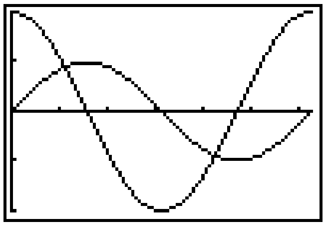
$a^2 + b^2 = 70$

The correct answer is 70.

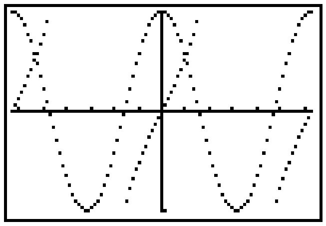
7-5 Solving Trigonometric Equations

Page 458 Graphing Calculator Exploration

1.



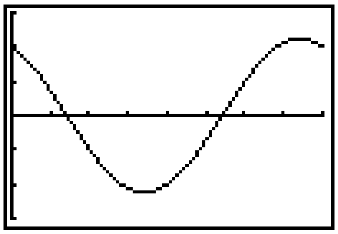
2.



3. Exercise 1: (1.1071, 0.8944), (4.2487, −0.8944)
Exercise 2: (−5.2872, 0.5437), (0.9960, 0.5437)

4. The x-coordinates are the solutions of the equations. Substitute the x-coordinates and see that the two sides of the equation are equal.

5.



$[0, 2\pi]$ sc1:$\frac{\pi}{4}$ by $[-3, 3]$ sc1:1

5a. The x-intercepts of the graph are the solutions of the equation $\sin x = 2 \cos x$. They are the same.

5b. $y = \tan 0.5x - \cos x$ or $y = \cos x - \tan 0.5x$

Page 459 Check for Understanding

1. A trigonometric identity is an equation that is true for all values of the variable for which each side of the equation is defined. A trigonometric equation that is not an identity is only true for certain values of the variable.

2. All trigonometric functions are periodic. Adding the least common multiple of the periods of the functions that appear to any solution to the equation will always produce another solution.

3. $45° + 360x°$ and $135° + 360x°$, where x is any integer

4. Each type of equation may require adding, subtracting, multiplying, or dividing each side by the same number. Quadratic and trigonometric equations can often be solved by factoring. Linear and quadratic equations do not require identities. All linear and quadratic equations can be solved algebraically, whereas some trigonometric equations require a graphing calculator. A linear equation has at most one solution. A quadratic equation has at most two solutions. A trigonometric equation usually has infinitely many solutions unless the values of the variable are restricted.

5. $2\sin x + 1 = 0$
$2\sin x = -1$
$\sin x = -\frac{1}{2}$
$x = -30°$

6. $2\cos x - \sqrt{3} = 0$
$2\cos x = \sqrt{3}$
$\cos x = \frac{\sqrt{3}}{2}$
$x = 30°$

7. $\sin x \cot x = \frac{\sqrt{3}}{2}$
$\sin x\left(\frac{\cos x}{\sin x}\right) = \frac{\sqrt{3}}{2}$
$\cos x = \frac{\sqrt{3}}{2}$
$x = 30°$ or $x = 330°$

8. $\cos 2x = \sin^2 x - 2$
$2\cos^2 x - 1 = (1 - \cos^2 x) - 2$
$2\cos^2 x - 1 = -\cos^2 x - 1$
$3\cos^2 x = 0$
$\cos^2 x = 0$
$\cos x = 0$
$x = 90°$ or $x = 270°$

9. $3\tan^2 x - 1 = 0$
$3\tan^2 x = 1$
$\tan^2 x = \frac{1}{3}$
$\tan x = \pm\frac{\sqrt{3}}{3}$
$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$ or $x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

10. $2\sin^2 x = 5\sin x + 3$
$2\sin^2 x - 5\sin x - 3 = 0$
$(2\sin x + 1)(\sin x - 3) = 0$
$2\sin x + 1 = 0$ or $\sin x - 3 = 0$
$\sin x = -\frac{1}{2}$ $\sin x = 3$
$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$ no solutions

11. $\sin^2 2x + \cos^2 x = 0$
$1 - \cos^2 2x + \cos^2 x = 0$
$1 - (2\cos^2 x - 1)^2 + \cos^2 x = 0$
$1 - (4\cos^4 x - 4\cos^2 x + 1) + \cos^2 x = 0$
$-4\cos^4 x + 5\cos^2 x = 0$
$\cos^2 x(-4\cos^2 x + 5) = 0$
$\cos^2 x = 0$ or $-4\cos^2 x + 5 = 0$
$\cos x = 0$ $\cos^2 x = \frac{5}{4}$
$x = \frac{\pi}{2} + \pi k$ $\cos x = \frac{\sqrt{5}}{2}$
no solutions

12. $\tan^2 x + 2\tan x + 1 = 0$
$(\tan x + 1)(\tan x + 1) = 0$
$\tan x + 1 = 0$
$\tan x = -1$
$x = \frac{3\pi}{4} + \pi k$

13. $\cos^2 x + 3\cos x = -2$
$\cos^2 x + 3\cos x + 2 = 0$
$(\cos x + 1)(\cos x + 2) = 0$
$\cos x + 1 = 0$ or $\cos x + 2 = 0$
$\cos x = -1$ $\cos x = -2$
$x = (2k + 1)\pi$ no solutions

14. $\sin 2x - \cos x = 0$
$2\sin x\cos x - \cos x = 0$
$\cos x(2\sin x - 1) = 0$
$\cos x = 0$ or $2\sin x - 1 = 0$
$x = \frac{\pi}{2} + k\pi$ $\sin x = \frac{1}{2}$
$x = \frac{\pi}{6} + 2\pi k$
or $x = \frac{5\pi}{6} + 2\pi k$

15. $2\cos\theta + 1 < 0$
$2\cos\theta < -1$
$\cos\theta < -\frac{1}{2}$
$\cos\theta = -\frac{1}{2}$ at $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$
$\frac{2\pi}{3} < \theta < \frac{4\pi}{3}$

16. $W = Fd\cos\theta$
$1500 = 100 \cdot 20\cos\theta$
$0.75 = \cos\theta$
$\theta \approx 41.41°$

Pages 459–461 Exercises

17. $\sqrt{2}\sin x - 1 = 0$
$\sqrt{2}\sin x = 1$
$\sin x = \frac{1}{\sqrt{2}}$
$\sin x = \frac{\sqrt{2}}{2}$
$x = 45°$

18. $2\cos x + 1 = 0$
$2\cos x = -1$
$\cos x = -\frac{1}{2}$
$x = 120°$

19. $\sin 2x - 1 = 0$
$2\sin x\cos x - 1 = 0$
$\sin^2 x\cos^2 x = \frac{1}{4}$
$\sin^2 x(1 - \sin^2 x) = \frac{1}{4}$
$\sin^2 x - \sin^4 x - \frac{1}{4} = 0$
$\sin^4 x - \sin^2 x + \frac{1}{4} = 0$
$\left(\sin^2 x - \frac{1}{2}\right)\left(\sin^2 x - \frac{1}{2}\right) = 0$
$\sin^2 x - \frac{1}{2} = 0$
$\sin^2 x = \frac{1}{2}$
$\sin x = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$
$x = 45°$

20. $\tan 2x - \sqrt{3} = 0$

$\tan 2x = \sqrt{3}$

$\frac{2\tan x}{1 - \tan^2 x} = \sqrt{3}$

$2\tan x = \sqrt{3}(1 - \tan^2 x)$

$2\tan x = \sqrt{3} - \sqrt{3}\tan^2 x$

$\sqrt{3}\tan^2 x + 2\tan x - \sqrt{3} = 0$

$(\sqrt{3}\tan x - 1)(\tan x + \sqrt{3}) = 0$

$\sqrt{3}\tan x - 1 = 0$ $\quad$ $\tan x + \sqrt{3} = 0$

$\tan x = \frac{1}{\sqrt{3}}$ $\quad$ $\tan x = -\sqrt{3}$

$\tan x = \frac{\sqrt{3}}{3}$ $\quad$ $x = -60°$

$x = 30°$

21. $\cos^2 x = \cos x$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$ or $\cos x - 1 = 0$

$x = 90°$ $\quad$ $\cos x = 1$

$x = 0°$

22. $\sin x = 1 + \cos^2 x$

$\sin x = 1 + 1 - \sin^2 x$

$\sin^2 x + \sin x - 2 = 0$

$(\sin x - 1)(\sin x + 2) = 0$

$\sin x - 1 = 0$ or $\sin x + 2 = 0$

$\sin x = 1$ $\quad$ $\sin x = -2$

$x = 90°$ $\quad$ no solution

23. $\sqrt{2}\cos x + 1 = 0$

$\sqrt{2}\cos x = -1$

$\cos x = -\frac{\sqrt{2}}{2}$

$x = 135°$ or $x = 225°$

24. $\cos x \tan x = \frac{1}{2}$

$\cos x \frac{\sin x}{\cos x} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = 30°$ or $x = 150°$

25. $\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0$ or $\tan x - 1 = 0$

$x = 0°$ or $x = 180°$ $\quad$ $\tan x = 1$

$x = 45°$ or $x = 225°$

26. $2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$2\cos x - 1 = 0$ or $\cos x + 2 = 0$

$2\cos x = 1$ $\quad$ $\cos x = -2$

$\cos x = \frac{1}{2}$ $\quad$ no solution

$x = 60°$ or $x = 300°$

27. $\sin 2x = -\sin x$

$2\sin x\cos x = -\sin x$

$2\sin x\cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$ or $2\cos x + 1 = 0$

$x = 0°$ or $x = 180°$ $\quad$ $2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = 120°$

or $x = 240°$

28. $\cos(x + 45°) + \cos(x - 45°) = \sqrt{2}$

$\cos x\cos 45° - \sin x\sin 45°$

$+ \cos x\cos 45° + \sin x\sin 45° = \sqrt{2}$

$\cos x \cdot \frac{\sqrt{2}}{2} - \sin x \cdot \frac{\sqrt{2}}{2}$

$+ \cos x \cdot \frac{\sqrt{2}}{2} + \sin x \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

$\sqrt{2}\cos x = \sqrt{2}$

$\cos x = 1$

$x = 0°$

29. $2\sin\theta\cos\theta + \sqrt{3}\sin\theta = 0$

$\sin\theta(2\cos\theta + \sqrt{3}) = 0$

$\sin\theta = 0$ or $2\cos\theta + \sqrt{3} = 0$

$\theta = 0°$ or $\theta = 180°$ $\quad$ $2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = 150°$

or $\theta = 210°$

30. $(2\sin x - 1)(2\cos^2 x - 1) = 0$

$2\sin x - 1 = 0$ or $2\cos^2 x - 1 = 0$

$2\sin x = 1$ $\quad$ $2\cos^2 x = 1$

$\sin x = \frac{1}{2}$ $\quad$ $\cos^2 x = \frac{1}{2}$

$x = \frac{\pi}{6}$ $\quad$ $\cos x = \pm\frac{\sqrt{2}}{2}$

or $x = \frac{5\pi}{6}$ $\quad$ $x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$

or $x = \frac{5\pi}{4}$ or $x = \frac{7\pi}{4}$

31. $4\sin^2 x + 1 = -4\sin x$

$4\sin^2 x + 4\sin x + 1 = 0$

$(2\sin x + 1)(2\sin x + 1) = 0$

$2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

32. $\sqrt{2}\tan x = 2\sin x$

$\sqrt{2}\frac{\sin x}{\cos x} = 2\sin x$

$\sqrt{2} = 2\cos x$

$\frac{\sqrt{2}}{2} = \cos x$

$x = \frac{\pi}{4}$ or $x = \frac{7\pi}{4}$

$\sqrt{2}\tan x = 2\sin x$ would also be true if both $\tan x$ and $\sin x$ equal 0. Since $\tan x = \frac{\sin x}{\cos x}$, $\tan x$ equals 0 when $\sin x = 0$. Therefore x can also equal 0 and π.

$0, \frac{\pi}{4}, \pi, \frac{7\pi}{4}$

33. $\sin x = \cos 2x - 1$

$\sin x = 1 - 2\sin^2 x - 1$

$2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0$ or $2\sin x + 1 = 0$

$x = 0$ or $x = \pi$ $\quad$ $2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ or

$x = \frac{11\pi}{6}$

34.
$$\begin{aligned}\cot^2 x - \csc x &= 1\\ \csc^2 x - 1 - \csc x &= 1\\ \csc^2 x - \csc x - 2 &= 0\\ (\csc x - 2)(\csc x + 1) &= 0\end{aligned}$$
$\csc x - 2 = 0$ or $\csc x + 1 = 0$

$\csc x = 2$ or $\csc x = -1$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -1$

$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$ $\quad$ $x = \frac{3\pi}{2}$

35.
$$\begin{aligned}\sin x + \cos x &= 0\\ \sin x &= -\cos x\\ \sin^2 x &= \cos^2 x\\ \sin^2 x - \cos^2 x &= 0\\ \sin^2 x - 1 + \sin^2 x &= 0\\ 2\sin^2 x - 1 &= 0\\ \sin^2 x &= \frac{1}{2}\\ \sin x &= \pm\frac{1}{\sqrt{2}} \text{ or } \pm\frac{\sqrt{2}}{2}\end{aligned}$$
$\sin x$ and $\cos x$ must be opposites, so $x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$.

36.
$$\begin{aligned}-1 - 3\sin\theta &= \cos 2\theta\\ -1 - 3\sin\theta &= 1 - 2\sin^2\theta\\ 2\sin^2\theta - 3\sin\theta - 2 &= 0\\ (2\sin\theta + 1)(\sin\theta - 2) &= 0\end{aligned}$$
$2\sin\theta + 1 = 0$ or $\sin\theta - 2 = 0$

$2\sin\theta = -1$ $\quad$ $\sin\theta = 2$

$\sin\theta = -\frac{1}{2}$ $\quad$ no solution

$\theta = \frac{7\pi}{6}$ or $\theta = \frac{11\pi}{6}$

37. $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6} + 2\pi k$ or $x = \frac{11\pi}{6} + 2\pi k$

38.
$$\begin{aligned}\cos x \tan x - 2\cos^2 x &= -1\\ \cos x \frac{\sin x}{\cos x} - 2\cos^2 x &= -1\\ \sin x - 2(1 - \sin^2 x) &= -1\\ 2\sin^2 x + \sin x - 1 &= 0\\ (2\sin x - 1)(\sin x + 1) &= 0\end{aligned}$$
$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = 1$ $\quad$ $\sin x = -1$

$\sin x = \frac{1}{2}$ $\quad$ $x = \frac{3\pi}{2} + 2\pi k$

$x = \frac{\pi}{6} + 2\pi k$ or $x = \frac{5\pi}{6} + 2\pi k$

39.
$$\begin{aligned}3\tan^2 x &= \sqrt{3}\tan x\\ 3\tan^2 x - \sqrt{3}\tan x &= 0\\ \tan x(3\tan x - \sqrt{3}) &= 0\end{aligned}$$
$\tan x = 0$ or $3\tan x - \sqrt{3} = 0$

$x = \pi k$ $\quad$ $3\tan x = \sqrt{3}$

$\tan x = \frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6} + \pi k$

40.
$$\begin{aligned}2(1 - \sin^2 x) &= 3\sin x\\ 2 - 2\sin^2 x &= 3\sin x\\ 2\sin^2 x + 3\sin x - 2 &= 0\\ (2\sin x - 1)(\sin x + 2) &= 0\end{aligned}$$
$2\sin x - 1 = 0$ or $\sin x + 2 = 0$

$2\sin x = 1$ $\quad$ $\sin x = -2$

$\sin x = \frac{1}{2}$ $\quad$ no solution

$x = \frac{\pi}{6} + 2\pi k$ or

$x = \frac{5\pi}{6} + 2\pi k$

41.
$$\begin{aligned}\frac{1}{\cos x - \sin x} &= \cos x + \sin x\\ (\cos x - \sin x)(\cos x + \sin x) &= 1\\ \cos^2 x - \sin^2 x &= 1\\ \cos^2 x - (1 - \cos^2 x) &= 1\\ 2\cos^2 x - 1 &= 1\\ 2\cos^2 x &= 2\\ \cos^2 x &= 1\\ \cos x &= \pm 1\\ x &= \pi k\end{aligned}$$

42.
$$\begin{aligned}2\tan^2 x - 3\sec x &= 0\\ 2(\sec^2 x - 1) - 3\sec x &= 0\\ (2\sec x + 1)(\sec x - 2) &= 0\\ 2\sec^2 x - 3\sec x - 2 &= 0\end{aligned}$$
$2\sec x + 1 = 0$ or $\sec x - 2 = 0$

$2\sec x = -1$ $\quad$ $\sec x = 2$

$\sec x = -\frac{1}{2}$ $\quad$ $\cos x = \frac{1}{2}$

$\cos x = -2$ $\quad$ $x = \frac{\pi}{3} + 2\pi k$ or

no solution $\quad$ $x = \frac{5\pi}{3} + 2\pi k$

43.
$$\begin{aligned}\sin x \cos x &= \frac{1}{2}\\ \sin^2 x \cos^2 x &= \frac{1}{4}\\ \sin^2 x(1 - \sin^2 x) &= \frac{1}{4}\\ \sin^2 x - \sin^4 x) &= \frac{1}{4}\\ \sin^4 x - \sin^2 x + \frac{1}{4} &= 0\\ \left(\sin^2 x - \frac{1}{2}\right)\left(\sin^2 x - \frac{1}{2}\right) &= 0\\ \sin^2 x - \frac{1}{2} &= 0\\ \sin^2 x &= \frac{1}{2}\\ \sin x &= \pm\frac{\sqrt{2}}{2}\\ x &= \frac{\pi}{4} + \pi k\end{aligned}$$

44.
$$\begin{aligned}\cos^2 x - \sin^2 x &= \frac{\sqrt{3}}{2}\\ \cos^2 x - (1 - \cos^2 x) &= \frac{\sqrt{3}}{2}\\ 2\cos^2 x - 1 &= \frac{\sqrt{3}}{2}\\ 2\cos^2 x &= \frac{2 + \sqrt{3}}{2}\\ \cos^2 x &= \frac{2 + \sqrt{3}}{4}\\ \cos x &= \pm\frac{\sqrt{2 + \sqrt{3}}}{2}\\ x &= \frac{\pi}{12} + \pi k \text{ or } x = \frac{11\pi}{12} + \pi k\end{aligned}$$

45.
$$\begin{aligned}\sin^4 x - 1 &= 0\\ (\sin^2 x - 1)(\sin^2 x + 1) &= 0\end{aligned}$$
$\sin^2 x - 1 = 0$ or $\sin^2 x + 1 = 0$

$\sin^2 x = 1$ $\quad$ $\sin^2 x = -1$

$\sin x = \pm 1$ $\quad$ no solutions

$x = \frac{\pi}{2} + \pi k$

46. $\sec^2 x + 2\sec x = 0$

$\sec x(\sec x + 2) = 0$

$\sec x = 0$ or $\sec x + 2 = 0$

$\cos x = \frac{1}{0}$ $\quad$ $\sec x = -2$

no solution $\quad$ $\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2\pi k$ or

$x = \frac{4\pi}{3} + 2\pi k$

47.
$$\begin{aligned}
\sin x + \cos x &= 1\\
\sin^2 x + 2\sin x\cos x + \cos^2 x &= 1\\
\sin^2 x + 2\sin x\cos x + 1 - \sin^2 x &= 1\\
2\sin x\cos x &= 0\\
\sin x\cos x &= 0\\
\sin^2 x\cos^2 x &= 0\\
\sin^2 x(1-\sin^2 x) &= 0
\end{aligned}$$
$\sin^2 x = 0$ or $1 - \sin^2 x = 0$

$\sin x = 0$ $\quad$ $\sin^2 x = 1$

$x = 2\pi k$ $\quad$ $\sin x = \pm 1$

$x = \frac{\pi}{2} + 2\pi k$

48.
$$\begin{aligned}
2\sin x + \csc x &= 3\\
2\sin^2 x + 1 &= 3\sin x\\
2\sin^2 x - 3\sin x + 1 &= 0\\
(2\sin x - 1)(\sin x - 1) &= 0
\end{aligned}$$
$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$2\sin x = 1$ $\quad$ $\sin x = 1$

$\sin x = \frac{1}{2}$ $\quad$ $x = \frac{\pi}{2} + 2\pi k$

$x = \frac{\pi}{6} + 2\pi k$ or

$x = \frac{5\pi}{6} + 2\pi k$

49. $\cos\theta \le -\frac{\sqrt{3}}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$ at $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

$\frac{5\pi}{6} \le \theta \le \frac{7\pi}{6}$

50. $\cos\theta - \frac{1}{2} > 0$

$\cos\theta > \frac{1}{2}$

$\cos\theta = \frac{1}{2}$ at $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

$0 \le \theta < \frac{\pi}{3}$ or $\frac{5\pi}{3} < \theta < 2\pi$

51. $\sqrt{2}\sin\theta - 1 < 0$

$\sqrt{2}\sin\theta < 1$

$\sin\theta < \frac{\sqrt{2}}{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$ at $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

$0 \le \theta < \frac{\pi}{4}$ or $\frac{3\pi}{4} < \theta < 2\pi$

52. 0.4636, 3.6052 **53.** 0, 1.8955

54. 0.3218, 3.4633

55. $\sin\theta = \frac{\lambda}{D}$

$\sin\theta = \frac{5.5 \times 10^{-7}}{0.003}$

$\sin\theta \approx 0.0001833333333$

$\theta \approx 0.01°$

56.
$$\begin{aligned}
\sin 2x &< \sin x\\
2\sin x\cos x &< \sin x\\
2\sin x\cos x - \sin x &< 0\\
\sin x(2\cos x - 1) &< 0
\end{aligned}$$
The product on the left side of the inequality is equal to 0 when x is 0, $\frac{\pi}{3}$, π, or $\frac{5\pi}{3}$. For the product to be negative, one factor must be positive and the other negative. This occurs if $\frac{\pi}{3} < x < \pi$ or $\frac{5\pi}{3} < x < 2\pi$.

57.
$$\begin{aligned}
R &= \frac{v^2}{g}\sin 2\theta\\
20 &= \frac{15^2}{9.8}\sin 2\theta
\end{aligned}$$
$0.8711111111 \approx \sin 2\theta$

$2\theta \approx 60.5880156$ or $2\theta \approx 119.4119844$

$\theta \approx 30.29°$ $\quad$ $\theta \approx 59.71°$

58a.
$$\begin{aligned}
n_1\sin i &= n_2\sin r\\
1.00\sin 35° &= 2.42\sin r\\
\sin r &= \frac{1.00\sin 35°}{2.42}\\
\sin r &\approx 0.2370150563\\
r &\approx 13.71°
\end{aligned}$$

58b. Measure the angles of incidence and refraction to determine the index of refraction. If the index is 2.42, the diamond is genuine.

59.
$$\begin{aligned}
D &= 0.5\sin(6.5x)\sin(2500t)\\
0.01 &= 0.5\sin(6.5(0.5))\sin(2500t)\\
0.02 &= \sin 3.25\sin 2500t\\
-0.1848511958 &\approx \sin 2500t\\
-0.1859549654 &\approx 2500t
\end{aligned}$$
The first positive angle with sine equivalent to $\sin(-0.1859549654)$ is $\pi + 0.1859549654$ or 3.326477773.

$t \approx \frac{3.326477773}{2500}$

$t \approx 0.0013$ s

60. $a\sin(bx + c) + d = d + \frac{a}{2}$

$a\sin(bx + c) = \frac{a}{2}$

$\sin(bx + c) = \frac{1}{2}$

The period of the function $\sin(bx + c)$ is $\frac{360°}{b}$, so the given interval consists of $\frac{360°}{\frac{360°}{b}} = b$ periods.

The equation $\sin(bx + c) = \frac{1}{2}$ has two solutions per period, so the total number of solutions is $2b$.

61. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} \sqrt{17} \\ 2\sqrt{2} \end{bmatrix}$$
$$\begin{bmatrix} 3\cos\theta - 4\sin\theta \\ 3\sin\theta + 4\cos\theta \end{bmatrix} = \begin{bmatrix} \sqrt{17} \\ 2\sqrt{2} \end{bmatrix}$$
$3\cos\theta - 4\sin\theta = \sqrt{17}$

$3\sin\theta + 4\cos\theta = 2\sqrt{2}$

$\downarrow$

$9\cos\theta - 12\sin\theta = 3\sqrt{17}$

$16\cos\theta + 12\sin\theta = 8\sqrt{2}$

$25\cos\theta = 8\sqrt{2} + 3\sqrt{17}$

$\cos\theta = \frac{8\sqrt{2} + 3\sqrt{17}}{25}$

$\theta \approx 18.68020037$

$360 - \theta \approx 341.32°$

62. $\cot 67.5° = \cot \frac{135°}{2}$ $\qquad \cot\theta = \frac{1}{\tan\theta}$

$\tan\frac{135°}{2} = \sqrt{\frac{1-\cos 135°}{1+\cos 135°}}$ (Quadrant 1)

$= \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{1+\left(-\frac{\sqrt{2}}{2}\right)}}$

$= \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}}}$

$= \sqrt{\frac{(2+\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}}$

$= \sqrt{\frac{(2+\sqrt{2})^2}{4-2}}$

$= \frac{2+\sqrt{2}}{\sqrt{2}}$

$\cot 67.5 = \frac{1}{\frac{2+\sqrt{2}}{\sqrt{2}}}$

$= \frac{\sqrt{2}}{2+\sqrt{2}}$

$= \frac{\sqrt{2}(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$

$= \frac{2\sqrt{2}-2}{4-2}$

$= \sqrt{2}-1$

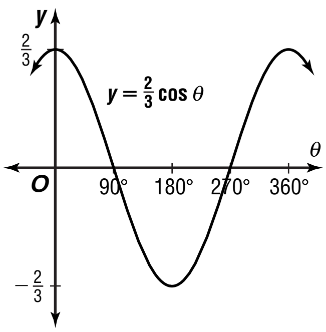
63. $\frac{\tan x}{\sec x} = \frac{\sqrt{2}}{5}$

$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sqrt{2}}{5}$

$\sin x = \frac{\sqrt{2}}{5}$

Sample answer: $\sin x = \frac{\sqrt{2}}{5}$

64. $A = \frac{2}{3}, 2\pi$



65. $\frac{45 \text{ miles}}{\text{hour}} \cdot \frac{5280 \text{ ft}}{\text{mile}} \cdot \frac{12 \text{ inches}}{\text{ft}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}} = 792$ in/sec

$v = r\frac{\theta}{t}$

$792 = 7\frac{\theta}{1}$

$\frac{792}{7} = \pi$

$\frac{792}{7}$ radians $\div 2\pi \approx 18$ rps

66. undefined

67.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & -2 \\ & & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$x^2 + 2x + 1 = 0$

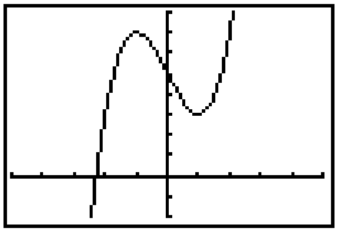
$(x+1)(x+1) = 0$

$x + 1 = 0 \qquad x + 1 = 0$

$x = -1 \qquad x = -1$

$(x-2)(x+1(x+1)$

68.



[−5, 5] sc1:1 by [−2, 8] sc1:1

max: (−1, 7), min: (1, 3)

69. $3x + 4 = 16 \qquad 6 = 2y$

$x = 4 \qquad y = 3 \qquad (4, 3)$

70.

$$\begin{array}{l} x - y + z = 1 \\ \underline{2x + y + 3z = 5} \\ 3x \quad + 4z = 6 \end{array} \qquad \begin{array}{l} x - y + z = 1 \\ \underline{x + y - z = 11} \\ 2x \quad = 12 \\ x = 6 \end{array}$$

$3x + 4z = 6 \qquad x + y - z = 11$

$3(6) + 4z = 6 \qquad 6 + y - (-3) = 11$

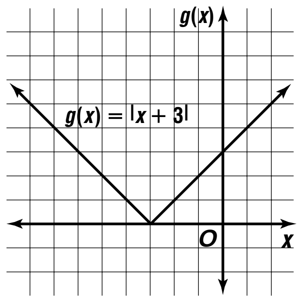
$4z = -12 \qquad y = 2$

$z = -3$

(6, 2, −3)

71.

x	$g(x)$
−7	4
−5	2
−3	0
−1	2
1	4



72. $A = \frac{1}{2}bh$

$A = \frac{1}{2}(6)(1)$

$A = 3$

The correct choice is C.

Page 462 History of Mathematics

1. $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 10°$
   $x \approx 0.87$
   $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 20°$
   $x \approx 1.74$
   $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 30°$
   $x \approx 2.59$
   $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 40°$
   $x \approx 3.42$
   $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 50°$
   $x \approx 4.23$
   $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 60°$
   $x = 5$
   $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 70°$
   $x \approx 5.74$
   $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 80°$
   $x \approx 6.43$
   $x^2 = 5^2 + 5^2 - 2(5)(5)\cos 90°$
   $x \approx 7.07$

Angle Measure	Length of Chord (cm)
10°	0.87
20°	1.74
30°	2.59
40°	3.42
50°	4.23
60°	5.00
70°	5.74
80°	6.43
90°	7.07

7-6 Normal Form of a Linear Equation

Page 467 Check for Understanding

1. *Normal* means perpendicular
2. Compute cos 30° and sin 30°. Use these as the coefficients of x and y, respectively, in the normal form. The normal form is $\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 10 = 0$.
3. The statement is true. The given line is tangent to the circle centered at the origin with radius p.
4.

Slope-Intercept Form: $y = mx + b$, displays slope and y-intercept
Point-Slope Form: $y - y_1 = m(x - x_1)$, displays slope and a point on the line
Standard Form: $Ax + by + C = 0$, displays no information
Normal Form: $x\cos\phi + y\sin\phi - p = 0$, displays length of the normal and the angle the normal makes with the x-axis

See students' work for sample problems.

5. $x\cos\phi + y\sin\phi - p = 0$
   $x\cos 30° + y\sin 30° - 10 = 0$
   $\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 10 = 0$
   $\sqrt{3}x + y - 20 = 0$
6. $x\cos\phi + y\sin\phi - p = 0$
   $x\cos 150° + y\sin 150° - \sqrt{3} = 0$
   $-\frac{\sqrt{3}}{2}x + \frac{1}{2}y - \sqrt{3} = 0$
   $\sqrt{3}x - y + 2\sqrt{3} = 0$
7. $x\cos\phi + y\sin\phi - p = 0$
   $x\cos\frac{7\pi}{4} + y\sin\frac{7\pi}{4} - 5\sqrt{2} = 0$
   $\frac{\sqrt{2}}{2}x + \left(-\frac{\sqrt{2}}{2}\right)y - 5\sqrt{2} = 0$
   $\sqrt{2}x - \sqrt{2}y - 10\sqrt{2} = 0$
   $x - y - 10 = 0$
8. $4x + 3y = -10$ $\quad -\sqrt{A^2 + B^2} = -\sqrt{4^2 + 3^2}$ or -5
   $4x + 3y + 10 = 0$ $\quad \frac{4}{-5}x + \frac{3}{-5}y + \frac{10}{-5} = 0$
   $-\frac{4}{5}x - \frac{3}{5}y - 2 = 0$
   $\sin\phi = -\frac{3}{5}$, $\cos\phi = -\frac{4}{5}$, $p = 2$; Quadrant III
   $\tan\phi = \frac{-\frac{3}{5}}{-\frac{4}{5}}$ or $\frac{3}{4}$
   ϕ
9. $y = -3x + 2$
   $3x + y - 2 = 0$
   $\sqrt{A^2 + B^2} = \sqrt{3^2 + 1^2}$ or $\sqrt{10}$
   $\frac{3}{\sqrt{10}}x + \frac{1}{\sqrt{10}}y - \frac{2}{\sqrt{10}} = 0$
   $\frac{3\sqrt{10}}{10}x + \frac{\sqrt{10}}{10}y - \frac{\sqrt{10}}{5} = 0$
   $\sin\phi = \frac{\sqrt{10}}{10}$, $\cos\phi = \frac{3\sqrt{10}}{10}$, $p = \frac{\sqrt{10}}{5}$; Quadrant I
   $\tan\phi = \frac{\frac{\sqrt{10}}{10}}{\frac{3\sqrt{10}}{10}}$ or $\frac{1}{3}$
   $\phi \approx 18°$

10. $\sqrt{2}x - \sqrt{2}y = 6$

$\sqrt{2}x - \sqrt{2}y - 6 = 0$

$\sqrt{A^2 + B^2} = \sqrt{\sqrt{2}^2 + (-\sqrt{2})^2}$ or 2

$\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - \frac{6}{2} = 0$

$\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - 3 = 0$

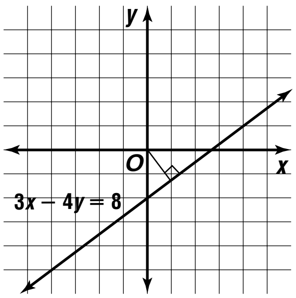
$\sin \phi = -\frac{\sqrt{2}}{2}$, $\cos \phi = \frac{\sqrt{2}}{2}$, $p = 3$; Quadrant IV

$\tan \phi = \dfrac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or -1

$\phi \approx 315°$

11a. $3x - 4y = 8$

$y = \frac{3}{4}x - 2$



11b. $3x - 4y = 8$

$3x - 4y - 8 = 0$

$\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2}$ or 5

$\frac{3}{5}x - \frac{4}{5}y - \frac{8}{5} = 0$

$p = \frac{8}{5}$ or 1.6 miles

Pages 467–469 Exercises

12. $x \cos \phi + y \sin \phi - p = 0$

$x \cos 60° + y \sin 60° - 15 = 0$

$\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 15 = 0$

$x + \sqrt{3}y - 30 = 0$

13. $x \cos \phi + y \sin \theta - p = 0$

$x \cos \frac{\pi}{4} + y \sin \frac{\pi}{4} - 12 = 0$

$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 12 = 0$

$\sqrt{2}x + \sqrt{2}y - 24 = 0$

14. $x \cos \phi + y \sin \phi - p = 0$

$x \cos 135° + y \sin 135° - 3\sqrt{2} = 0$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 3\sqrt{2} = 0$

$-\sqrt{2}x + \sqrt{2}y - 6\sqrt{2} = 0$

$x - y + 6 = 0$

15. $x \cos \phi + y \sin \phi - p = 0$

$x \cos \frac{5\pi}{6} + y \sin \frac{5\pi}{6} - 2\sqrt{3} = 0$

$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 2\sqrt{3} = 0$

$\sqrt{3}x - y + 4\sqrt{3} = 0$

16. $x \cos \phi + y \sin \phi - p = 0$

$x \cos \frac{\pi}{2} + y \sin \frac{\pi}{2} - 2 = 0$

$0x + 1y - 2 = 0$

$y - 2 = 0$

17. $x \cos \phi + y \sin \phi - p = 0$

$x \cos 210° + y \sin 210° - 5 = 0$

$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y - 5 = 0$

$\sqrt{3}x + y + 10 = 0$

18. $x \cos \phi + y \sin \phi - p = 0$

$x \cos \frac{4\pi}{3} + y \sin \frac{4\pi}{3} - 5 = 0$

$-\frac{1}{2}x - \frac{\sqrt{3}}{2}y - 5 = 0$

$x + \sqrt{3}y + 10 = 0$

19. $x \cos \phi + y \sin \phi - p = 0$

$x \cos 300° + y \sin 300° - \frac{3}{2} = 0$

$\frac{1}{2}x - \frac{\sqrt{3}}{2}y - \frac{3}{2} = 0$

$x - \sqrt{3}y - 3 = 0$

20. $x \cos \phi + y \sin \phi - p = 0$

$x \cos \frac{11\pi}{6} + y \sin \frac{11\pi}{6} - 4\sqrt{3} = 0$

$\frac{\sqrt{3}}{2}x - \frac{1}{2}y - 4\sqrt{3} = 0$

$\sqrt{3}x - y - 8\sqrt{3} = 0$

21. $-\sqrt{A^2 + B^2} = -\sqrt{5^2 + 12^2}$ or -13

$\frac{5}{-13}x + \frac{12}{-13}y + \frac{65}{-13} = 0$

$-\frac{5}{13}x - \frac{12}{13}y - 5 = 0$

$\sin\phi = -\frac{12}{13}$, $\cos \phi = -\frac{5}{13}$, $p = 5$; Quadrant III

$\tan \phi = \dfrac{-\frac{12}{13}}{-\frac{5}{13}}$ or $\frac{12}{5}$

$\phi \approx 247°$

22. $x + y = 1$

$x + y - 1 = 0$

$\sqrt{A^2 + B^2} = \sqrt{1^2 + 1^2}$ or $\sqrt{2}$

$\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{1}{\sqrt{2}} = 0$

$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2} = 0$

$\sin \phi = \frac{\sqrt{2}}{2}$, $\cos \phi = \frac{\sqrt{2}}{2}$, $p = \frac{\sqrt{2}}{2}$; Quadrant I

$\tan \phi = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ or 1

$\phi = 45°$

23. $3x - 4y = 15$

$3x - 4y - 15 = 0$

$\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2}$ or 5

$\frac{3}{5}x - \frac{4}{5}y - \frac{15}{5} = 0$

$\frac{3}{5}x - \frac{4}{5}y - 3 = 0$

$\sin \phi = -\frac{4}{5}$, $\cos \phi = \frac{3}{5}$, $p = 3$; Quadrant IV

$\tan \phi = \dfrac{-\frac{4}{5}}{\frac{3}{5}}$ or $-\frac{4}{3}$

$\phi \approx 307°$

24. $y = 2x - 4$

$-2x + y + 4 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{(-2)^2 + 1^2}$ or $-\sqrt{5}$

$-\frac{2}{-\sqrt{5}}x + \frac{1}{-\sqrt{5}}y + \frac{4}{-\sqrt{5}} = 0$

$\frac{2\sqrt{5}}{5}x - \frac{\sqrt{5}}{5}y - \frac{4\sqrt{5}}{5} = 0$

$\sin \phi = -\frac{\sqrt{5}}{5}, \cos \phi = \frac{2\sqrt{5}}{5}, p = \frac{4\sqrt{5}}{5}$; Quadrant IV

$\tan \phi = \frac{-\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}}$ or $-\frac{1}{2}$

$\phi \approx 333°$

25. $x = 3$

$x - 3 = 0$

$\sqrt{A^2 + B^2} = \sqrt{1^2 + 0^2}$ or 1

$\frac{1}{1}x - \frac{3}{1} = 0$

$x - 3 = 0$

$\sin \phi = 0, \sin \phi = 1, p = 3$

$\tan \phi = \frac{0}{1}$ or 0

$\phi = 0°$

26. $-\sqrt{3}x - y = 2$

$-\sqrt{3}x - y - 2 = 0$

$\sqrt{A^2 + B^2} = \sqrt{(-\sqrt{3})^2 + (-1)^2}$ or 2

$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y - \frac{2}{2} = 0$

$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y - 1 = 0$

$\sin \phi = -\frac{1}{2}, \cos \phi = -\frac{\sqrt{3}}{2}, p = 1$; Quadrant III

$\tan \phi = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$ or $\frac{\sqrt{3}}{3}$

$\phi \approx 210°$

27. $y - 2 = \frac{1}{4}(x + 20)$

$y - 2 = \frac{1}{4}x + 5$

$-x + 4y - 28 = 0$

$\sqrt{A^2 + B^2} = \sqrt{(-1)^2 + 4^2}$ or $\sqrt{17}$

$-\frac{1}{\sqrt{17}}x + \frac{4}{\sqrt{17}}y - \frac{28}{\sqrt{17}} = 0$

$-\frac{\sqrt{17}}{17}x + \frac{4\sqrt{17}}{17}y - \frac{28\sqrt{17}}{17} = 0$

$\sin \phi = \frac{4\sqrt{17}}{17}, \cos \phi = -\frac{\sqrt{17}}{17}, p = \frac{28\sqrt{17}}{17}$; Quadrant II

$\tan \phi = \frac{\frac{4\sqrt{17}}{17}}{-\frac{\sqrt{17}}{17}}$ or -4

$\phi \approx 104°$

28. $\frac{x}{3} = y - 4$

$\frac{x}{3} - y + 4 = 0$

$x - 3y + 12 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{1^2 + (-3)^2}$ or $-\sqrt{10}$

$\frac{1}{-\sqrt{10}}x - \frac{3}{-\sqrt{10}}y + \frac{12}{-\sqrt{10}} = 0$

$-\frac{\sqrt{10}}{10}x + \frac{3\sqrt{10}}{10}y - \frac{6\sqrt{10}}{5} = 0$

$\sin \phi = \frac{3\sqrt{10}}{10}, \cos \phi = -\frac{\sqrt{10}}{10}, p = \frac{6\sqrt{10}}{5}$; Quadrant II

$\tan \phi = \frac{\frac{3\sqrt{10}}{10}}{-\frac{\sqrt{10}}{10}}$ or -3

$\phi \approx 108°$

29. $\frac{x}{20} + \frac{y}{24} = 1$

$\frac{x}{20} + \frac{y}{24} - 1 = 0$

$6x + 5y - 124 = 0$

$\sqrt{A^2 + B^2} = \sqrt{6^2 + 5^2}$ or $\sqrt{61}$

$\frac{6}{\sqrt{61}}x + \frac{5}{\sqrt{61}}y - \frac{124}{\sqrt{61}} = 0$

$\frac{6\sqrt{61}}{61}x + \frac{5\sqrt{61}}{61}y - \frac{120\sqrt{61}}{61} = 0$

$\sin \phi = \frac{5\sqrt{61}}{61}, \cos \phi = \frac{6\sqrt{61}}{61}, p = \frac{120\sqrt{61}}{61}$; Quadrant I

$\tan \phi = \frac{\frac{5\sqrt{61}}{61}}{\frac{6\sqrt{61}}{61}}$ or $\frac{5}{6}$

$\phi \approx 40°$

30. $\sqrt{A^2 + B^2} = \sqrt{6^2 + 8^2}$ or 10; $p = 10$

$\cos \phi = \frac{6}{10}$ or $\frac{3}{5}$, $\sin \phi = \frac{8}{10}$ or $\frac{4}{5}$

$x \cos \phi + y \sin \phi - p = 0$

$\frac{3}{5}x + \frac{4}{5}y - 10 = 0$

$3x + 4y - 50 = 0$

31. $\sqrt{A^2 + B^2} = \sqrt{(-4)^2 + 4^2}$ or $4\sqrt{2}$; $p = 4\sqrt{2}$

$\cos \phi = \frac{-4}{4\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\sin \phi = \frac{4}{4\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$x \cos \phi + y \sin \phi - p = 0$

$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 4\sqrt{2} = 0$

$x - y + 8 = 0$

32. $2\sqrt{2}x = y + 18$

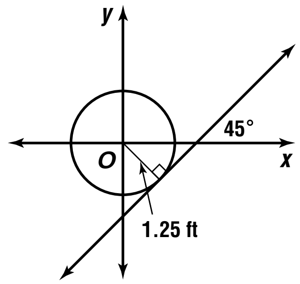
$2\sqrt{2}x - y - 18 = 0$

$\sqrt{A^2 + B^2} = \sqrt{2(2)^2 + (-1)^2} = \sqrt{9} = 3$

$\frac{2\sqrt{2}}{3}x - \frac{1}{3}y - \frac{18}{3} = 0$

$p = \frac{18}{3} = 6$ units

33a.



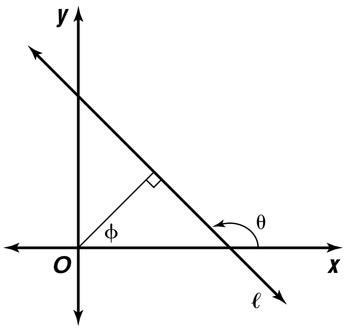
33b. $p = 1.25, \phi = 45°$

$x \cos(-45°) + y \sin(-45°) - 1.25 = 0$

$\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - 1.25 = 0$

$\sqrt{2}x - \sqrt{2}y - 2.5 = 0$

34a.



ϕ and the supplement of θ are complementary angles of a right triangle, so $\phi + 180° - \theta = 90°$. Simplifying this equation gives $\theta = \phi + 90°$.

34b. $\tan \theta$. The slope of a line is the tangent of the angle the line makes with the positive x-axis

34c. Since the normal line is perpendicular to ℓ, the slope of the normal line is the negative reciprocal of the slope of ℓ. That is, $-\frac{1}{\tan \theta} = -\cot \theta$.

34d. The slope of ℓ is the negative reciprocal of the slope of the normal, or $-\frac{1}{\tan \phi} = -\cot \phi$.

35a. $\sqrt{A^2 + B^2} = \sqrt{5^2 + 12^2}$ or 13

$\frac{5}{13}x + \frac{12}{13}y - \frac{39}{13} = 0$

$\frac{5}{13}x + \frac{12}{13}y - 3 = 0$

35b. $\sin \phi = \frac{12}{13}, \cos \phi = \frac{5}{13}$; Quadrant I

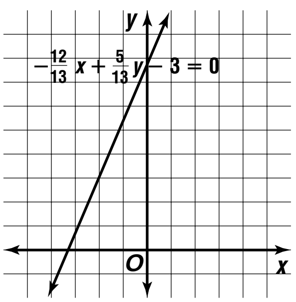
$\tan \phi = \frac{\frac{12}{13}}{\frac{5}{13}}$ or $\frac{12}{5}$

$\phi \approx 67°$

$\phi + 90° = 67° + 90°$ or $157°$

$x \cos 157° + y \sin 157° - 3 = 0$

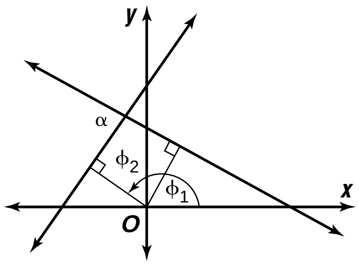
$-\frac{12}{13}x + \frac{5}{13}y - 3 = 0$



35c. See students' work.

35d. The line with normal form $x \cos \phi + y \sin \phi - p = 0$ makes an angle of ϕ with the positive x-axis and has a normal of length p. The graph of Armando's equation is a line whose normal makes an angle of $\phi + \delta$ with the x-axis and also has length p. Therefore, the graph of Armando's equation is the graph of the original line rotated $\delta°$ counterclockwise about the origin. Armando is correct. See students' graphs.

36a.



The angles of the quadrilateral are $180° - \alpha$, $90°$, $\phi_2 - \phi_1$, and $90°$. Then $180° - \alpha + 90° + \phi_2 - \phi_1 + 90° = 360°$, which simplifies to $\phi_2 = \phi_1 + \alpha$. If the lines intersect so that α is an interior angle of the quadrilateral, the equation works out to be $\phi_2 = 180° + \phi_1 - \alpha$.

36b. $\tan \phi_2 = \tan(\phi_1 + \alpha)$

$= \frac{\tan \phi_1 + \tan \alpha}{1 - \tan \phi_1 \tan \alpha}$

If the lines intersect so that α is an interior angle of the quadrilateral, the equation works out to be $\tan \phi_2 = \frac{\tan \phi_1 - \tan \alpha}{1 + \tan \phi_1 \tan \alpha}$.

37. $5x - y = 15$

$5x - y - 15 = 0$

$\sqrt{A^2 + B^2} = \sqrt{5^2 + (-1)^2}$ or $\sqrt{26}$

$\frac{5}{\sqrt{26}}x - \frac{1}{\sqrt{26}}y - \frac{15}{\sqrt{26}} = 0$

$\frac{5\sqrt{26}}{26}x - \frac{\sqrt{26}}{26}y - \frac{15\sqrt{26}}{26} = 0, p = \frac{15\sqrt{26}}{26}$

$3x + 4y = 36$

$3x + 4y - 36 = 0$

$\sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2}$ or 5

$\frac{3}{5}x + \frac{4}{5}y - \frac{36}{5} = 0, p = \frac{36}{5}$

$5x - 2y = -20$

$5x - 2y + 20 = 0$

$\sqrt{5^2 + (-2)^2} = \sqrt{25 + 4}$ or $\sqrt{29}$

$\frac{5}{\sqrt{29}}x - \frac{2}{\sqrt{29}}y + \frac{20}{\sqrt{29}} = 0$

$\frac{5\sqrt{29}}{29}x - \frac{2\sqrt{29}}{29}y + \frac{20\sqrt{29}}{29} = 0, p = \frac{20\sqrt{29}}{29}$

$\frac{15\sqrt{26}}{26} + \frac{36}{5} + \frac{20\sqrt{29}}{29} \approx 13.85564879$

$13.85564879 \times 500 \approx 6927.824395$; \$6927.82

38. $2\cos^2 x + 7\cos x - 4 = 0$

$(2\cos x - 1)(\cos x + 4) = 0$

$2\cos x - 1 = 0$ or $\cos x + 4 = 0$

$2\cos x = 1$ $\quad$ $\cos x = -4$

$\cos x = \frac{1}{2}$ $\quad$ no solution

$x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

39. $\sin x = \sqrt{1 - \cos^2 x}$ $\qquad$ $\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{1}{6}\right)^2}$ $\qquad$ $= \sqrt{1 - \left(\frac{2}{3}\right)^2}$

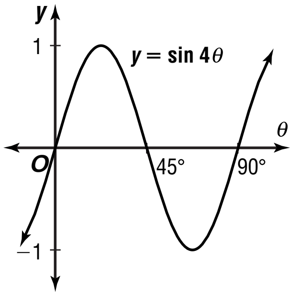
$= \sqrt{\frac{36}{36}}$ or $\frac{\sqrt{35}}{6}$ $\qquad$ $= \sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{\sqrt{35}}{6}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{6}\right)\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{2\sqrt{35} + \sqrt{5}}{18}$

40. $A = 1, \frac{2\pi}{4} = \frac{\pi}{2}$ or 90°



41. $r = \frac{d}{2}$

$r = \frac{13.4}{2}$ or 6.7

$x^2 = 6.7^2 + 6.7^2 - 2(6.7)(6.7) \cos 26°20'$

$x^2 \approx 9.316604344$

$x \approx 3.05$ cm

42.

$$\frac{x}{x-5} + \frac{17}{25-x^2} = \frac{1}{x+5}$$
$$\frac{x}{x-5} + \frac{-17}{x^2-25} = \frac{1}{x+5}$$
$$(x-5)(x+5)\left(\frac{x}{x-5}\right) +$$
$$(x-5)(x+5)\left(\frac{-17}{(x-5)(x+5)}\right) = (x-5)(x+5)\left(\frac{1}{x+5}\right)$$
$$x(x+5) - 17 = x - 5$$
$$x^2 + 5x - 17 = x - 5$$
$$x^2 + 4x - 12 = 0$$
$$(x+6)(x-2) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -6 \qquad\qquad x = 2$$

43. original box: $V = \ell wh$

$= 4 \cdot 6 \cdot 2$

$= 48$

new box: $V = \ell wh$

$1.5(48) = (4 + x)(6 + x)(2 + x)$

$72 = x^3 + 12x^2 + 44x + 48$

$0 = x^3 + 12x^2 + 44x - 24$

x	$V(x)$
0.4	−4.416
0.5	1.125

$V(0.5)$ is closer to zero, so $x = 0.5$.

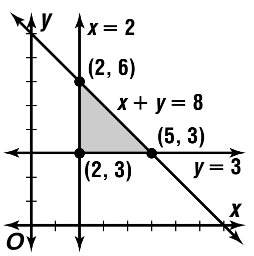
$4 + x = 4 + 0.5$ or 4.5

$6 + x = 6 + 0.5$ or 6.5

$2 + x = 2 + 0.5$ or 2.5

4.5 in. by 6.5 in. by 2.5 in.

44.



$f(x, y) = 3x - y + 4$

$f(2, 3) = 3(2) - 3 + 4$ or 7

$f(2, 6) = 3(2) - 6 + 4$ or 4

$f(5, 3) = 3(5) - 3 + 4$ or 16

16, 4

45.
$$\frac{1}{\begin{vmatrix}-1 & 2\\ 4 & 3\end{vmatrix}}\begin{bmatrix}3 & -2\\ 4 & -1\end{bmatrix} = \frac{1}{5}\begin{bmatrix}3 & -2\\ 4 & -1\end{bmatrix}$$
$$\begin{bmatrix}-1 & 2\\ -4 & 3\end{bmatrix} \cdot \begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}0\\ 15\end{bmatrix}$$
$$\frac{1}{5}\begin{bmatrix}3 & -2\\ 4 & -1\end{bmatrix} \cdot \begin{bmatrix}-1 & 2\\ -4 & 3\end{bmatrix} \cdot \begin{bmatrix}x\\ y\end{bmatrix} = \frac{1}{5}\begin{bmatrix}3 & -2\\ 4 & -1\end{bmatrix} \cdot \begin{bmatrix}0\\ 15\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix} = \frac{1}{5}\begin{bmatrix}-30\\ -15\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-6\\ -3\end{bmatrix}$$

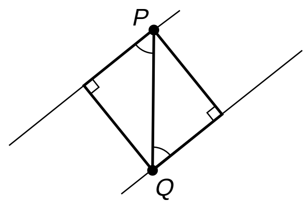
$(-6, -3)$

46. The value of $2a + b$ cannot be determined from the given information. The correct choice is E.

7-7 Distance From a Point to a Line

Page 474 Check for Understanding

1. The distance from a point to a line is the distance from that point to the closest point on the line.
2. The sign should be chosen opposite the sign of C where $Ax + By + C = 0$ is the standard form of the equation of the line.
3. In the figure, P and Q are any points on the lines. The right triangles are congruent by AAS. The corresponding congruent sides of the triangles show that the same distance is always obtained between the two lines.



4. The formula is valid in either case. Examples will vary. For a vertical line, $x = a$, the formula subtracts a from the x-coordinate of the point. For a horizontal line, $y = b$, the formula subtracts b from the y-coordinate of the point.
5. $2x - 3y = -2 \rightarrow 2x - 3y + 2 = 0$

$$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$$
$$d = \frac{2(1) + (-3)(2) + 2}{-\sqrt{2^2 + (-3)^2}}$$
$$d = \frac{-2}{-\sqrt{13}} \text{ or } \frac{2\sqrt{13}}{13}$$

6. $6x - y = -3 \rightarrow 6x - y + 3 = 0$

$$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$$
$$d = \frac{6(-2) + (-1)(3) + 3}{-\sqrt{6^2 + (-1)^2}}$$
$$d = \frac{-12}{-\sqrt{37}} \text{ or } \frac{12\sqrt{37}}{37}$$

7. $3x - 5y = 1$ When $x = 2$, $y = 1$. Use (2, 1).
$3x - 5y = -3 \rightarrow 3x - 5y + 3 = 0$
$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$
$d = \frac{3(2) + (-5)(1) + 3}{-\sqrt{3^2 + (-5)^2}}$
$d = \frac{4}{-\sqrt{34}}$ or $-\frac{2\sqrt{34}}{17}$
$\frac{2\sqrt{34}}{17}$

8. $y = -\frac{1}{3}x + 3$ Use (0, 3).
$y = -\frac{1}{3}x - 7 \rightarrow x + 3y + 21 = 0$
$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$
$d = \frac{1(0) + 3(3) + 21}{-\sqrt{1^2 + 3^2}}$
$d = \frac{30}{-\sqrt{10}}$ or $-3\sqrt{10}$
$3\sqrt{10}$

9. $d_1 = \frac{6x_1 + 8y_1 + 5}{\sqrt{6^2 + 8^2}}$ $d_2 = \frac{2x_1 - 3y_1 - 4}{\sqrt{2^2 + (-3)^2}}$
$\frac{6x_1 + 8y_1 + 5}{10} = \frac{2x_1 - 3y_1 - 4}{\sqrt{13}}$
$6\sqrt{13}x + 8\sqrt{13}y + 5\sqrt{13} = 20x - 30y - 40$
$(20 - 6\sqrt{13})x -$
$(30 + 8\sqrt{13})y - 40 - 5\sqrt{13} = 0;$
$\frac{6x_1 + 8y_1 + 5}{10} = -\frac{2x_1 - 3y_1 = 4}{\sqrt{13}}$
$6\sqrt{13}x + 8\sqrt{13}y + 5\sqrt{13} = -20x + 30y + 40$
$(20 + 6\sqrt{13})x + (8\sqrt{13} - 30)y - 40 + 5\sqrt{13} = 0$

10. (2000, 0)
$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$
$d = \frac{5(2000) + (-3)(0) + 0}{\sqrt{5^2 + (-3)^2}}$
$d = \frac{10{,}000}{\sqrt{34}}$ or about 1715 ft

Pages 475–476 Exercises

11. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$
$d = \frac{3(2) + (-4)(0) + 15}{-\sqrt{3^2 + (-4)^2}}$
$d = \frac{21}{-5}$
$\frac{21}{5}$

12. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$
$d = \frac{5(3) + (-3)(5) + 10}{-\sqrt{5^2 + (-3)^2}}$
$d = \frac{10}{-\sqrt{34}}$ or $-\frac{5\sqrt{34}}{17}$
$\frac{5\sqrt{34}}{17}$

13. $-2x - y = -3 \rightarrow -2x - y + 3 = 0$
$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$
$d = \frac{-2(0) + (-1)(0) + 3}{\sqrt{(-2)^2 + (-1)^2}}$
$d = \frac{3}{\sqrt{5}}$ or $\frac{3\sqrt{5}}{5}$

14. $y = 4 - \frac{2}{3}x \rightarrow 2x + 3y - 12 = 0$
$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$
$d = \frac{2(-2) + 3(-3) + (-12)}{\sqrt{2^2 + 3^2}}$
$d = \frac{-25}{\sqrt{13}}$ or $-\frac{25\sqrt{13}}{13}$
$\frac{25\sqrt{13}}{13}$

15. $y = 2x - 5 \rightarrow 2x - y - 5 = 0$
$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$
$d = \frac{2(3) + 3(-1)(1) + (-5)}{\sqrt{2^2 + (-1)^2}}$
$d = \frac{0}{\sqrt{5}}$ or 0

16. $y = -\frac{4}{3}x + 6 \rightarrow 4x + 3y - 18 = 0$
$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$
$d = \frac{4(-1) + 3(2) + (-18)}{\sqrt{4^2 + 3^2}}$
$d = \frac{-16}{5}$ or $-\frac{16}{5}$
$\frac{16}{5}$

17. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$
$d = \frac{3(0) + (-1)(0) + 1}{-\sqrt{3^2 + (-1)^2}}$
$d = \frac{1}{-\sqrt{10}}$ or $-\frac{\sqrt{10}}{10}$
$\frac{\sqrt{10}}{10}$

18. $6x - 8y = 3$ When $x = 0$, $y = -\frac{3}{8}$. Use $\left(0, -\frac{3}{8}\right)$.
$6x - 8y = -5 \rightarrow 6x - 8y + 5 = 0$
$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$
$d = \frac{6(0) + (-8)\left(-\frac{3}{8}\right) + 1}{-\sqrt{6^2 + (-8)^2}}$
$d = \frac{8}{-10}$ or $-\frac{4}{5}$
$\frac{4}{5}$

19. $4x - 5y = 12$ When $x = 3$, $y = 0$. Use (3, 0).
$4x - 5y = 6 \rightarrow 4x - 5y - 6 = 0$
$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$
$d = \frac{4(3) + (-5)(0) + (-6)}{\sqrt{4^2 + (-5)^2}}$
$d = \frac{6}{\sqrt{41}}$ or $\frac{6\sqrt{41}}{41}$

20. $y = 2x + 1$ Use (0, 1).
$2x - y = 2 \rightarrow 2x - y - 2 = 0$
$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$
$d = \frac{2(0) + (-1)(1) + (-2)}{\sqrt{2^2 + (-1)^2}}$
$d = \frac{-3}{\sqrt{5}}$ or $-\frac{3\sqrt{5}}{5}$
$\frac{3\sqrt{5}}{5}$

21. $y = -3x + 6$ Use (0, 6).

$3x + y = 4 \rightarrow 3x + y - 4 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{3(0) + 1(6)(1) + (-4)}{\sqrt{3^2 + 1^2}}$

$d = \frac{2}{\sqrt{10}}$ or $\frac{\sqrt{10}}{5}$

22. $y = \frac{8}{5}x - 1$ Use (0, −1).

$8x + 15 = 5y \rightarrow 8x - 5y + 15 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{8(0) + (-5)(-1) + 15}{-\sqrt{8^2 + (-5)^2}}$

$d = \frac{20}{-\sqrt{89}}$ or $-\frac{20\sqrt{89}}{89}$

$\frac{20\sqrt{89}}{89}$

23. $y = -\frac{3}{2}x$ Use (0, 0).

$y = -\frac{3}{2}x - 4 \rightarrow 3x + 2y + 8 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{3(0) + 2(0) + 8}{-\sqrt{3^2 + 2^2}}$

$d = \frac{8}{-\sqrt{13}}$ or $-\frac{8\sqrt{13}}{13}$

$\frac{8\sqrt{13}}{13}$

24. $y = -x + 6$ Use (0, 6).

$x + y - 1 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{1(0) + 1(6) + (-1)}{\sqrt{1^2 + 1^2}}$

$d = \frac{5}{\sqrt{2}}$ or $\frac{5\sqrt{2}}{2}$

25. $d_1 = \frac{3x_1 + 4y_1 - 10}{\sqrt{3^2 + 4^2}}$ $d_2 = \frac{5x_1 - 12y_1 - 26}{\sqrt{5^2 + (-12)^2}}$

$\frac{3x_1 + 4y_1 - 10}{5} = \frac{5x_1 - 12y_1 - 26}{13}$

$39x + 52y - 130 = 25x - 60y - 130$

$14x + 112y = 0$

$x + 8y = 0$

$\frac{3x_1 + 4y_1 - 10}{5} = -\frac{5x_1 - 12y_1 - 26}{13}$

$39x + 52y - 130 = -25x + 60y + 130$

$64x - 8y - 260 = 0$

$16x - 2y - 65 = 0$

26. $d_1 = \frac{4x_1 + y_1 - 6}{\sqrt{4^2 + 1^2}}$ $d_2 = \frac{-15x_1 + 8y_1 - 68}{\sqrt{(-15)^2 + 8^2}}$

$\frac{4x_1 + y_1 - 6}{\sqrt{17}} = -\frac{-15x_1 + 8y_1 - 68}{17}$

$68x + 17y - 102 = -15\sqrt{12}x + 8\sqrt{17}y - 68\sqrt{17}$

$(68 + 15\sqrt{17})x + (17 - 8\sqrt{17})y - 102 + 68\sqrt{17} = 0$

$\frac{4x_1 + y_1 - 6}{\sqrt{17}} = -\frac{-15x_1 + 8y_1 - 68}{17}$

$68x + 17y - 102 = 15\sqrt{12}x - 8\sqrt{17}y + 68\sqrt{17}$

$(68 - 15\sqrt{17})x + (17 + 8\sqrt{17})y - 102 - 68\sqrt{17} = 0$

27. $y = \frac{2}{3}x + 1 \rightarrow 2x - 3y + 3 = 0$

$y = -3x - 2 \rightarrow 3x + y + 2 = 0$

$d_1 = \frac{2x_1 - 3y_1 + 3}{-\sqrt{2^2 + (-3)^2}}$ $d_2 = \frac{3x_1 - y_1 + 2}{-\sqrt{3^2 + 1^2}}$

$\frac{2x_1 - 3y_1 + 3}{-\sqrt{13}} = -\frac{3x_1 + y_1 + 2}{-\sqrt{10}}$

$2\sqrt{10}x - 3\sqrt{10}y + 3\sqrt{10} = -3\sqrt{13}x - \sqrt{13}y - 2\sqrt{13}$

$(2\sqrt{10} + 3\sqrt{13})x + (\sqrt{13} - 3\sqrt{10})y + 3\sqrt{10} + 2\sqrt{13} = 0$

$\frac{2x_1 - 3y_1 + 3}{-\sqrt{13}} = \frac{-3x_1 + y_1 + 2}{-\sqrt{10}}$

$-2\sqrt{10}x + 3\sqrt{10}y - 3\sqrt{10} = -3\sqrt{13}x - \sqrt{13}y - 2\sqrt{13}$

$(-2\sqrt{10} + 3\sqrt{13})x + (\sqrt{13} + 3\sqrt{10})y - 3\sqrt{10} + 2\sqrt{13} = 0$

28a. Linda: (19, 112)

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{4(19) + (-3)(112) + 228}{-\sqrt{4^2 + (-3)^2}}$

$d = \frac{-32}{-5}$ or 6.4

Father: (45, 120)

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{4(45) + (-3)(120) + 228}{-\sqrt{4^2 + (-3)^2}}$

$d = \frac{48}{-5}$ or −9.6

Linda

28b. $4x - 3y + 228 = 0$

$4x - 3(140) + 228 = 0$

$4x = 192$

$x = 48$

29. Let $x = 1$.

$\tan\theta = \frac{y}{x}$

$\tan 40° = \frac{y}{1}$

$y \approx 0.8390996312$

$m = \frac{0.839 - 0}{1 - 0}$

$m \approx 0.839$

$y - y_1 = m(x - x_1)$

$y - 0.839 \approx 0.839(x - 1)$

$y \approx 0.839x$

$-0.839x + y \approx 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d \approx \frac{-0.839(16) + 1(12) + 0}{\sqrt{0.839^2 + 1^2}}$

$d \approx -1.092068438$

1.09 m

30. The radius of the circle is $\sqrt{[(-5) - (-2)]^2 + (6 - 2)^2}$ or 5. Now find the distance from the center of the circle to the line.

$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$

$d = \frac{5(-5) + (-12)(6) + 32}{-\sqrt{5^2 + (-12)^2}}$

$d = \frac{-65}{-13}$

$d = 5$

Since the distance from the center of the circle to the line is the same as the radius of the circle, the line can only intersect the circle in one point. That is, the line is tangent to the circle.

31. $m_1 = \frac{4-7}{-3-1}$ or $\frac{3}{4}$

$y - 7 = \frac{3}{4}(x - 1)$

$3x - 4y + 25 = 0$

$a_1 = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$a_1 = \frac{3(-1) + (-4)(-3) + 25}{-\sqrt{3^2 + (-4)^2}}$

$a_1 = -\frac{34}{5}$

$m_2 = \frac{-3-4}{-1-(-3)}$ or $-\frac{7}{2}$

$y - 4 = -\frac{7}{2}(x - (-3))$

$7x + 2y + 13 = 0$

$a_2 = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$a_2 = \frac{7(1) + 2(7) + 13}{-\sqrt{7^2 + 2^2}}$

$a_2 = \frac{34}{-\sqrt{53}}$ or $-\frac{34\sqrt{53}}{53}$

$m_3 = \frac{7-(-3)}{1-(-1)}$ or 5

$y - 7 = 5(x - 1)$

$5x - y + 2 = 0$

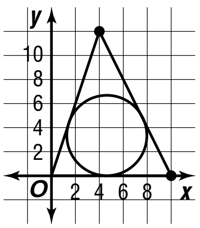
$a_3 = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$a_3 = \frac{5(-3) + (-1)(4) + 2}{-\sqrt{5^2 + (-1)^2}}$

$a_3 = \frac{-17}{-\sqrt{26}}$ or $\frac{17\sqrt{26}}{26}$

$\frac{34}{5}, \frac{34\sqrt{53}}{53}, \frac{17\sqrt{26}}{26}$

32.



The standard form of the equation of the line through (0, 0) and (4, 12) is $3x - y = 0$. The standard form of the equation of the line through (4, 12) and (10, 0) is $2x + y - 20 = 0$. The standard form for the x-axis is $y = 0$. To find the bisector of the angle at the origin, set $\frac{3x - y}{\sqrt{10}} = y$ and solve to obtain $y = \frac{3}{1 + \sqrt{10}}x$. To find the bisector of the angle of the triangle at (10, 0), set $\frac{2x + y - 20}{\sqrt{5}} = -y$ and solve to obtain $2x + (1 + \sqrt{5})y - 20 = 0$. The intersection of these two bisectors is the center of the inscribed circle. To solve the system of equations, substitute $y = \frac{3}{1 + \sqrt{10}}x$ into the equation of the other bisector and solve for x to get $x = \frac{20(1 + \sqrt{10})}{5 + 3\sqrt{5} + 2\sqrt{10}}$. Then $y = \frac{20(1 + \sqrt{10})}{5 + 3\sqrt{5} + 2\sqrt{10}} \cdot \frac{3}{1 + \sqrt{10}} = \frac{60}{5 + 3\sqrt{5} + 2\sqrt{10}}$. This y-coordinate is the inradius of the triangle. The approximate value is 3.33.

33. $-2x + 7y = 5$

$2x - 7y + 5 = 0$

$-\sqrt{A^2 + B^2} = -\sqrt{2^2 + (-7)^2}$ or $-\sqrt{53}$

$\frac{2}{-\sqrt{53}}x - \frac{7}{-\sqrt{53}}y + \frac{5}{-\sqrt{53}} = 0$

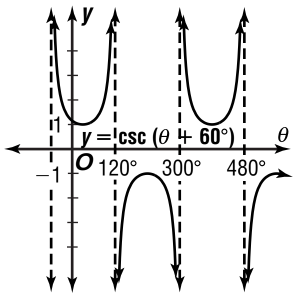
$-\frac{2\sqrt{53}}{53}x + \frac{7\sqrt{53}}{53}y - \frac{5\sqrt{53}}{53} = 0$

34. $\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(\frac{\sqrt{3}}{6}\right)^2$

$= \frac{5}{6}$

35. $\frac{2\pi}{1} = 2\pi, \frac{60°}{1} = 60°$



36. $110 - 3 = 330 \qquad 180° - (60° + 40°) = 80°$

$x^2 = 330^2 + 330^2 - 2(330)(330)\cos 80°$

$x^2 \approx 179979.4269$

$x \approx 424.24$ miles

37. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T = 2\pi\sqrt{\frac{2}{9.8}}$

$T \approx 2.8$ s

38.

2	1	8	k
		2	20
	1	10	$20 + k$

$20 + k = 0$

$k = -20$

39. $2x + y - z = -9$

$2(-x + 3y - 2z) = 2(10)$ →

$$\begin{aligned} 2x + y - z &= -9 \\ -2x + 6y - 4z &= 20 \\ \hline 7y - 5z &= 11 \end{aligned}$$

$$\begin{aligned} x - 2y + z &= -7 \\ -x + 3y - 2z &= 10 \\ \hline y - z &= 3 \end{aligned}$$

$-5(y - z) = -5(3)$

$7y - 5z = 11$ →

$$\begin{aligned} -5y + 5z &= -15 \\ 7y - 5z &= 11 \\ \hline 2y &= -4 \\ y &= -2 \end{aligned}$$

$y - z = 3$

$-2 - z = 3$

$-5 = z$

$x - 2y + z = -7$

$x - 2(-2) + (-5) = -7$

$x = -6$

$(-6, -2, -5)$

40. square: $A = s^2$

$16 = s^2$

$4 = s$

triangle: $A = \frac{1}{2}bh$

$6 = \frac{1}{2}(4)h$

$3 = h$

$AE = s + h$

$AE = 4 + 3$ or 7

$EF = AE$

$EF = 7$

The correct choice is C.

Chapter 7 Study Guide and Assessment

Page 477 Understanding and Using the Vocabulary

1. b **2.** g **3.** d **4.** a
5. i **6.** j **7.** h **8.** f
9. e **10.** c

Pages 478–480 Skills and Concepts

11. $$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{2}} = 2$$

12. $$\tan^2\theta + 1 = \sec^2\theta$$
$$4^2 + 1 = \sec^2\theta$$
$$17 = \sec^2\theta$$
$$\sqrt{17} = \sec\theta$$

13. $$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{16}{25}$$
$$\cos\theta = \frac{4}{5}$$

14. $$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\tan^2\theta + 1 = \left(\frac{5}{4}\right)^2$$
$$\tan^2\theta = \frac{9}{16}$$
$$\tan\theta = \frac{3}{4}$$

15. $$\csc x - \cos^2 x\csc x = \frac{1}{\sin x} - (1 - \sin^2 x)\left(\frac{1}{\sin x}\right) = \frac{1}{\sin x} - \frac{1}{\sin x} + \sin x = \sin x$$

16. $$\cos^2 x + \tan^2 x\cos^2 x \stackrel{?}{=} 1$$
$$\cos^2 x + \frac{\sin^2 x}{\cos^2 x}\cos^2 x \stackrel{?}{=} 1$$
$$\cos^2 x + \sin^2 x = 1$$
$$1 = 1$$

17. $$\frac{1-\cos\theta}{1+\cos\theta} \stackrel{?}{=} (\csc\theta - \cot\theta)^2$$
$$\frac{1-\cos\theta}{1+\cos\theta} \stackrel{?}{=} \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$
$$\frac{1-\cos\theta}{1+\cos\theta} \stackrel{?}{=} \frac{(1-\cos\theta)^2}{\sin^2\theta}$$
$$\frac{1-\cos\theta}{1+\cos\theta} \stackrel{?}{=} \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$
$$\frac{1-\cos\theta}{1+\cos\theta} \stackrel{?}{=} \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$
$$\frac{1-\cos\theta}{1+\cos\theta} \stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta}$$

18. $$\frac{\sec\theta+1}{\tan\theta} \stackrel{?}{=} \frac{\tan\theta}{\sec\theta - 1}$$
$$\frac{\sec\theta+1}{\tan\theta} \stackrel{?}{=} \frac{\tan\theta(\sec\theta+1)}{\sec^2\theta - 1}$$
$$\frac{\sec\theta+1}{\tan\theta} \stackrel{?}{=} \frac{\tan\theta(\sec\theta+1)}{\tan^2\theta}$$
$$\frac{\sec\theta+1}{\tan\theta} = \frac{\sec\theta+1}{\tan\theta}$$

19. $$\frac{\sin^4 x - \cos^4 x}{\sin^2 x} \stackrel{?}{=} 1 - \cot^2 x$$
$$\frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\sin^2 x} \stackrel{?}{=} 1 - \cot^2 x$$
$$\frac{\sin^2 x - \cos^2 x}{\sin^2 x} \stackrel{?}{=} 1 - \cot^2 x$$
$$1 - \frac{\cos^2 x}{\sin^2 x} \stackrel{?}{=} 1 - \cot^2 x$$
$$1 - \cot^2 x = 1 - \cot^2 x$$

20. $$\cos 195° = \cos(150° + 45°)$$
$$= \cos 150°\cos 45° - \sin 150°\sin 45°$$
$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{-\sqrt{6} - \sqrt{2}}{4}$$

21. $$\cos 15° = \cos(45° - 30°)$$
$$= \cos 45°\cos 30° + \sin 45°\sin 30°$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

22. $$\sin\left(-\frac{17\pi}{12}\right) = -\sin\frac{17\pi}{12}$$
$$= -\sin\left(\frac{\pi}{4} + \frac{7\pi}{6}\right)$$
$$= -\left(\sin\frac{\pi}{4}\cos\frac{7\pi}{6} + \cos\frac{\pi}{4}\sin\frac{7\pi}{6}\right)$$
$$= -\left(\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)\right)$$
$$= -\left(\frac{-\sqrt{6} - \sqrt{2}}{4}\right)$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

23. $$\tan\frac{11\pi}{12} = \tan\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$$
$$= \frac{\tan\frac{2\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{2\pi}{3}\tan\frac{\pi}{4}}$$
$$= \frac{-\sqrt{3}+1}{1 - (-\sqrt{3})(1)}$$
$$= \frac{1-\sqrt{3}}{1+\sqrt{3}}$$
$$= \frac{4 - 2\sqrt{3}}{-2} \text{ or } -2 + \sqrt{3}$$

24. $$\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{7}{25}\right)^2} = \sqrt{\frac{576}{625}} \text{ or } \frac{24}{25}$$
$$\sin y = \sqrt{1 - \cos^2 x} = \sqrt{1 - \left(\frac{2}{3}\right)^2} = \sqrt{\frac{5}{9}} \text{ or } \frac{\sqrt{5}}{3}$$
$$\cos(x - y) = \cos x\cos y + \sin x\sin y$$
$$= \left(\frac{24}{25}\right)\left(\frac{2}{3}\right) + \left(\frac{7}{25}\right)\left(\frac{\sqrt{5}}{3}\right)$$
$$= \frac{48 + 7\sqrt{5}}{75}$$

25. $\cos y = \frac{1}{\sec y}$

$= \frac{1}{\frac{3}{2}}$

$= \frac{2}{3}$

$\sin y = \sqrt{1 - \cos^2 y}$

$= \sqrt{1 - \left(\frac{2}{3}\right)^2}$

$= \sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$

$\tan y = \frac{\sin y}{\cos y}$

$= \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}$ or $\frac{\sqrt{5}}{2}$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$= \frac{\frac{5}{4} + \frac{\sqrt{5}}{2}}{1 - \left(\frac{5}{4}\right)\left(\frac{\sqrt{5}}{2}\right)}$

$= \frac{\frac{5 + 2\sqrt{5}}{4}}{\frac{8 - 5\sqrt{5}}{8}}$

$= \frac{10 + 4\sqrt{5}}{8 - 5\sqrt{5}}$

$= \frac{180 + 82\sqrt{5}}{-61}$ or $-\frac{180 + 25\sqrt{5}}{61}$

26. $\cos 75° = \cos \frac{150°}{2}$

$= \sqrt{\frac{1 + \cos 150°}{2}}$ (Quadrant I)

$= \sqrt{\frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2}}$

$= \frac{\sqrt{2 - \sqrt{3}}}{2}$

27. $\sin \frac{7\pi}{8} = \sin \frac{\frac{7\pi}{4}}{2}$

$= \sqrt{\frac{1 - \cos \frac{7\pi}{4}}{2}}$ (Quadrant II)

$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$

$= \frac{\sqrt{2 - \sqrt{2}}}{2}$

28. $\sin 22.5° = \sin \frac{45°}{2}$

$= \sqrt{\frac{1 - \cos 45°}{2}}$ (Quadrant I)

$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$

$= \frac{\sqrt{2 - \sqrt{2}}}{2}$

29. $\tan \frac{\pi}{12} = \tan \frac{\frac{\pi}{6}}{2}$

$= \sqrt{\frac{1 - \cos \frac{\pi}{6}}{1 + \cos \frac{\pi}{6}}}$ (Quadrant I)

$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}}$

$= \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}}$

$= \sqrt{\frac{(2 - \sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}}$

$= \sqrt{\frac{(2 - \sqrt{3})^2}{4 - 3}}$

$= 2 - \sqrt{3}$

30. $\sin^2 \theta + \cos^2 \theta = 1$

$\sin^2 \theta + \left(\frac{3}{5}\right)^2 = 1$

$\sin^2 \theta = \frac{16}{25}$

$\sin \theta = \frac{4}{5}$

$\sin 2\theta = 2 \sin \theta \cos \theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \frac{24}{25}$

31. $\cos 2\theta = 2 \cos^2 \theta - 1$

$= 2\left(\frac{3}{5}\right)^2 - 1$

$= -\frac{7}{25}$

32. $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$= \frac{\frac{4}{5}}{\frac{3}{5}}$ or $\frac{4}{3}$

$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

$= \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2}$

$= -\frac{24}{7}$

33. $\sin 4\theta = \sin 2(2\theta)$

$= 2 \sin 2\theta \cos 2\theta$

$= 2\left(\frac{24}{25}\right)\left(-\frac{7}{25}\right)$

$= -\frac{336}{625}$

34.

$\tan x + 1 = \sec x$

$(\tan x + 1)^2 = \sec^2 x$

$\tan^2 x + 2 \tan x + 1 = \tan^2 x + 1$

$2 \tan x = 0$

$\tan x = 0$

$x = 0°$

35.

$\sin^2 x + \cos 2x - \cos x = 0$

$1 - \cos^2 x + 2 \cos^2 x - 1 - \cos x = 0$

$\cos^2 x - \cos x = 0$

$\cos x (\cos x - 1) = 0$

$\cos x = 0$ or $\cos x - 1 = 0$

$x = 90°$ or $x = 270°$

$\cos x = 1$

$x = 0°$

36. $\cos 2x + \sin x = 1$
$1 - 2\sin^2 x + \sin x = 1$
$2\sin^2 x - \sin x = 0$
$\sin x(2\sin x - 1) = 0$
$\sin x = 0$ or $2\sin x - 1 = 0$
$x = 0°$ or $x = 180°$ $\quad \sin x = \frac{1}{2}$
$x = 30°$ or
$x = 150°$

37. $\sin x \tan x - \frac{\sqrt{2}}{2}\tan x = 0$
$\tan x\left(\sin x - \frac{\sqrt{2}}{2}\right) = 0$
$\tan x = 0$ or $\sin x - \frac{\sqrt{2}}{2} = 0$
$x = \pi k$ $\quad \sin x = \frac{\sqrt{2}}{2}$
$x = \frac{\pi}{4} + 2\pi k$ or $\frac{3\pi}{4} + 2\pi k$

38. $\sin 2x + \sin x = 0$
$2\sin x\cos x + \sin x = 0$
$\sin x(2\cos x + 1) = 0$
$\sin x = 0$ or $2\cos x + 1 = 0$
$x = \pi k$ $\quad \cos x = -\frac{1}{2}$
$x = \frac{2\pi}{3} + 2\pi k$
or $x = \frac{4\pi}{3} + 2\pi k$

39. $\cos^2 x = 2 - \cos x$
$\cos^2 x + \cos x - 2 = 0$
$(\cos x - 1)(\cos x + 2) = 0$
$\cos x - 1 = 0$ or $\cos x + 2 = 0$
$\cos x = 1$ $\quad \cos x = -2$
$x = 2\pi k$ $\quad$ no solution

40. $x\cos\phi + y\sin\phi - p = 0$
$x\cos\frac{\pi}{3} + y\sin\frac{\pi}{3} - 2\sqrt{3} = 0$
$\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 2\sqrt{3} = 0$
$x + \sqrt{3}y - 4\sqrt{3} = 0$

41. $x\cos\phi + y\sin\phi - p = 0$
$x\cos 90° + y\sin 90° - 5 = 0$
$0x + 1y - 5 = 0$
$y - 5 = 0$

42. $x\cos\phi + y\sin\phi - p = 0$
$x\cos\frac{2\pi}{3} + y\sin\frac{2\pi}{3} - 3 = 0$
$-\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 3 = 0$
$-x + \sqrt{3}y - 6 = 0$

43. $x\cos\phi + y\sin\phi - p = 0$
$x\cos 225° + y\sin 225° - 4\sqrt{2} = 0$
$-\frac{\sqrt{2}}{2}x + \left(-\frac{\sqrt{2}}{2}\right)y - 4\sqrt{2} = 0$
$x + y + 8 = 0$

44. $\sqrt{A^2 + B^2} = \sqrt{7^2 + 3^2}$ or $\sqrt{58}$
$\frac{7}{\sqrt{58}}x + \frac{3}{\sqrt{58}}y - \frac{8}{\sqrt{58}} = 0$
$\frac{7\sqrt{58}}{58}x + \frac{3\sqrt{58}}{58}y - \frac{4\sqrt{58}}{29} = 0$
$\sin\phi = \frac{3\sqrt{58}}{58}$, $\cos\phi = \frac{7\sqrt{58}}{58}$, $p = \frac{4\sqrt{58}}{29}$; Quadrant I
$\tan\phi = \dfrac{\frac{3\sqrt{58}}{58}}{\frac{7\sqrt{58}}{58}}$ or $\frac{3}{7}$
$\phi \approx 23°$

45. $6x = 4y - 5$
$6x - 4y + 5 = 0$
$-\sqrt{A^2 + B^2} = -\sqrt{6^2 + (-4)^2}$ or $-2\sqrt{13}$
$\frac{6}{-2\sqrt{13}}x - \frac{4}{-2\sqrt{13}}y + \frac{5}{-2\sqrt{13}} = 0$
$-\frac{3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y - \frac{5\sqrt{13}}{26} = 0$
$\sin\phi = \frac{2\sqrt{13}}{13}$, $\cos\phi = -\frac{3\sqrt{13}}{13}$, $p = \frac{5\sqrt{13}}{26}$; Quadrant II
$\tan\phi = \dfrac{\frac{2\sqrt{13}}{13}}{-\frac{3\sqrt{13}}{13}}$ or $-\frac{2}{3}$
$\phi \approx 146°$

46. $9x = -5y + 3$
$9x + 5y - 3 = 0$
$\sqrt{A^2 + B^2} = \sqrt{9^2 + 5^2}$ or $\sqrt{106}$
$\frac{9}{\sqrt{106}}x + \frac{5}{\sqrt{106}}y - \frac{3}{\sqrt{106}} = 0$
$\frac{9\sqrt{106}}{106}x + \frac{5\sqrt{106}}{106}y - \frac{3\sqrt{106}}{106} = 0$
$\sin\phi = \frac{5\sqrt{106}}{106}$, $\cos\phi = \frac{9\sqrt{106}}{106}$, $p = \frac{3\sqrt{106}}{106}$; Quadrant I
$\tan\phi = \dfrac{106}{\frac{9\sqrt{106}}{106}}$ or $\frac{5}{9}$
$\phi \approx 29°$

47. $x - 7y = -5$
$x - 7y + 5 = 0$
$-\sqrt{A^2 + B^2} = -\sqrt{1^2 + (-7)^2}$ or $-5\sqrt{2}$
$\frac{1}{-5\sqrt{2}}x - \frac{7}{-5\sqrt{2}}y + \frac{5}{-5\sqrt{2}} = 0$
$-\frac{\sqrt{2}}{10}x + \frac{7\sqrt{2}}{10}y - \frac{\sqrt{2}}{2} = 0$
$\sin\phi = \frac{7\sqrt{2}}{10}$, $\cos\phi = -\frac{\sqrt{2}}{10}$, $p = \frac{\sqrt{2}}{2}$; Quadrant II
$\tan\phi = \dfrac{\frac{7\sqrt{2}}{10}}{-\frac{\sqrt{2}}{10}}$ or -7
$\phi \approx 98°$

48. $d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$
$d = \frac{2(5) + (-3)(6) + 2}{-\sqrt{2^2 + (-3)^2}}$
$d = \frac{-6}{-\sqrt{13}}$ or $\frac{6\sqrt{13}}{13}$

49. $2y = -3x + 6 \rightarrow 3x + 2y - 6 = 0$
$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$
$d = \frac{3(-3) + 2(-4) + (-6)}{\sqrt{3^2 + 2^2}}$
$d = \frac{-23}{\sqrt{13}}$ or $-\frac{23\sqrt{13}}{13}$
$\frac{23\sqrt{13}}{13}$

50. $4y = 3x - 1 \rightarrow 3x - 4y - 1 = 0$
$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$
$d = \frac{3(-2) + (-4)(4) + (-1)}{\sqrt{3^2 + (-4)^2}}$
$d = \frac{-23}{5}$ or $-\frac{23}{5}$
$\frac{23}{5}$

51. $y = \frac{1}{3}x + 6 \rightarrow x - 3y + 18 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{1(21) + (-3)(20) + 18}{-\sqrt{1^2 + (-3)^2}}$

$d = \frac{-21}{-\sqrt{10}}$ or $\frac{21\sqrt{10}}{10}$

52. $y = \frac{x}{3} - 6$ Use $(0, -6)$.

$y = \frac{x}{3} + 2 \rightarrow x - 3y + 6 = 0$

$d = \frac{Ax_1 + By_1 + C}{-\sqrt{A^2 + B^2}}$

$d = \frac{1(0) + (-3)(-6) + 6}{-\sqrt{1^2 + (-3)^2}}$

$d = \frac{24}{-\sqrt{10}}$ or $-\frac{12\sqrt{10}}{5}$

$d = \frac{12\sqrt{10}}{5}$

53. $y = \frac{3}{4}x + 3$ Use $(0, 3)$.

$y = \frac{3}{4}x - \frac{1}{2} \rightarrow 3x - 4y - 2 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{3(0) + (-4)(3) + (-2)}{\sqrt{3^2 + (-4)^2}}$

$d = \frac{-14}{5}$ or $-\frac{14}{5}$

$d = \frac{14}{5}$

54. $x + y = 1$ Use $(0, 1)$.

$x + y = 5 \rightarrow x + y - 5 = 0$

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{1(0) + 1(1) + (-5)}{\sqrt{1^2 + 1^2}}$

$d = \frac{-4}{\sqrt{2}}$ or $-2\sqrt{2}$

$d = 2\sqrt{2}$

55. $y = \frac{2}{3}x - 2$ Use $(0, -2)$.

$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{2(0) + (-3)(-2) + 3}{\sqrt{2^2 + (-3)^2}}$

$d = \frac{9}{\sqrt{13}}$ or $\frac{9\sqrt{13}}{13}$

56. $y = -3x - 2 \rightarrow 3x + y + 2 = 0$

$y = -\frac{x}{2} + \frac{3}{2} \rightarrow x + 2y - 3 = 0$

$d_1 = \frac{3x_1 + y_1 + 2}{-\sqrt{3^2 + 1^2}}$ $d_2 = \frac{x_1 + 2y_1 - 3}{\sqrt{1^2 + 2^2}}$

$\frac{3x_1 + y_1 + 2}{-\sqrt{10}} = \frac{x_1 + 2y_1 - 3}{\sqrt{5}}$

$3\sqrt{5}x + \sqrt{5}y + 2\sqrt{5} = -\sqrt{10}x - 2\sqrt{10}y + 3\sqrt{10}$

$(3\sqrt{5} + \sqrt{10})x + (\sqrt{5} + 2\sqrt{10})y + 2\sqrt{5} - 3\sqrt{10} = 0$

$\frac{3x_1 + y_1 + 2}{-\sqrt{10}} = -\frac{x_1 + 2y_1 - 3}{\sqrt{5}}$

$3\sqrt{5}x + \sqrt{5}y + 2\sqrt{5} = \sqrt{10}x + 2\sqrt{10}y - 3\sqrt{10}$

$(3\sqrt{5} - \sqrt{10})x + (\sqrt{5} - 2\sqrt{10})y + 2\sqrt{5} + 3\sqrt{10} = 0$

57. $-x + 3y - 2 = 0$

$y = \frac{3}{5}x + 3 \rightarrow 3x - 5y + 15 = 0$

$d_1 = \frac{-x_1 + 3y_1 - 2}{\sqrt{(-1)^2 + 3^2}}$ $d_2 = \frac{3x_1 - 5y_1 + 15}{-\sqrt{3^2 + (-5)^2}}$

$\frac{-x_1 + 3y_1 - 2}{\sqrt{10}} = -\frac{3x_1 - 5y_1 + 15}{-\sqrt{34}}$

$-\sqrt{34}x + 3\sqrt{34}y - 2\sqrt{34} = 3\sqrt{10}x - 5\sqrt{10}y + 15\sqrt{10}$

$(-\sqrt{34} - 3\sqrt{10})x + (3\sqrt{34} + 5\sqrt{10})y - 2\sqrt{34} - 15\sqrt{10} = 0$

$\frac{-x_1 + 3y_1 - 2}{\sqrt{10}} = \frac{3x_1 - 5y_1 + 15}{-\sqrt{34}}$

$3\sqrt{10}x - 5\sqrt{10}y + 15\sqrt{10} = \sqrt{34}x - 3\sqrt{34}y + 2\sqrt{34}$

$(-\sqrt{34} + 3\sqrt{10})x + (3\sqrt{34} - 5\sqrt{10})y - 2\sqrt{34} + 15\sqrt{10} = 0$

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58. The formulas are equivalent.

$$\frac{v_0^2 \tan^2 \theta}{2g \sec^2 \theta} = \frac{v_0^2 \cdot \frac{\sin^2 \theta}{\cos^2 \theta}}{2g \cdot \frac{1}{\cos^2 \theta}}$$

$$= \frac{v_0^2 \cdot \frac{\sin^2 \theta}{\cos^2 \theta}}{2g \cdot \frac{1}{\cos^2 \theta}} \cdot \frac{\cos^2 \theta}{\cos^2 \theta}$$

$$= \frac{v_0^2 \sin^2 \theta}{2g}$$

59. $d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$

$d = \frac{4(1600) + (-2)(0) + 0}{\sqrt{4^2 + (-2)^2}}$

$d = \frac{6400}{\sqrt{20}}$

$d = 1431$ ft

60. $\sin 30° = \frac{x}{100}$ $30° + 45° + \theta = 90°$

$100 \sin 30° = x$ $\theta = 15°$

$50 = x$

$\cos \theta = \frac{x}{y}$

$\cos 15° = \frac{50}{y}$

$y = \frac{50}{\cos 15°}$

$y \approx 51.76$ yd

Page 481 Open-Ended Assessment

1. Sample answer: $15°$; $15° = \frac{30°}{2}$

$$\sin \frac{30°}{2} = \sqrt{\frac{1 - \cos 30°}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$\cos \frac{30°}{2} = \sqrt{\frac{1 + \cos 30°}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$\tan \frac{30°}{2} = \sqrt{\frac{1 - \cos 30°}{1 + \cos 30°}}$$
$$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}}$$
$$= \sqrt{\frac{\frac{2 - \sqrt{3}}{2}}{\frac{2 + \sqrt{3}}{2}}}$$
$$= \sqrt{\frac{(2 - \sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}}$$
$$= \sqrt{\frac{(2 - 3)^2}{4 - 3}}$$
$$= 2 - \sqrt{3}$$

2. Sample answer: $\sin x \tan x = \frac{1 - \cos^2 x}{\cos x}$

$$\sin x \tan x = \frac{1 - \cos^2 x}{\cos x}$$
$$\sin x \frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos x}$$
$$\frac{\sin^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$

SAT & ACT Preparation

Page 483 SAT and ACT Practice

1. The problem states that the measure of $\angle A$ is 80°. Since the measure of $\angle B$ is half the measure of $\angle A$, the measure of $\angle B$ must be 40°. Because $\angle A$, $\angle B$, and $\angle C$ are interior angles of a triangle, the sum of their measures must equal 180°.

$$m\angle A + m\angle B + m\angle C = 180$$
$$80 + 40 + m\angle C = 180$$
$$120 + m\angle C = 180$$
$$m\angle C = 60$$

The correct choice is B.

2. To find the point of intersection, you need to solve a system of two linear equations. Substitution or elimination by addition or subtraction can be used to solve a system of equations. To solve this system of equations, use substitution. Substitute $2x - 2$ for y in the second equation.

$$7x - 3y = 11$$
$$7x - 3(2x - 2) = 11$$
$$7x - 6x + 6 = 11$$
$$x = 5$$

Then use this value for x to calculate the value for y.

$y = 2x - 2$
$y = 2(5) - 2$ or 8

The point of intersection is (5, 8). The correct choice is A.

3. One way to solve this problem is to label the three interior angles of the triangle, a, b, and c. Then write equations using these angles and the exterior angles.

$a + b + c = 180$
$x + a = 180$
$y + b = 180$
$z + c = 180$

Add the last three equations.

$x + a + y + b + z + c = 180 + 180 + 180$
$x + y + z + a + b + c = 180 + 180 + 180$

Replace $a + b + c$ with 180.

$x + y + z + 180 = 180 + 180 + 180$
$x + y + z = 180 + 180$ or 360

The correct choice is D.

4. Since $x + y = 90°$, $x = 90° - y$.

Then $\sin x = \sin (90° - y)$.

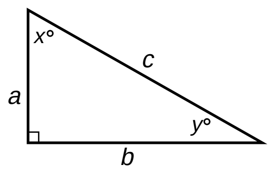
$\sin (90° - y) = \cos y$

$$\frac{\sin x}{\cos y} = \frac{\sin(90° - y)}{\cos y} = \frac{\cos y}{\cos y} = 1$$

The correct choice is D.

Another solution is to draw a diagram and notice that $\sin x = \frac{b}{c}$ and $\cos y = \frac{b}{c}$.

$$\frac{\sin x}{\cos y} = \frac{\frac{b}{c}}{\frac{b}{c}} = 1$$



5. In order to represent the slopes, you need the coordinates of point A. Since A lies on the y-axis, let its coordinates be $(0, y)$. Then calculate the two slopes. The slope of $\overline{AB}$ is $\frac{y - 0}{0 - (-3)} = \frac{y}{3}$. The slope of $\overline{AD}$ is $\frac{y - 0}{0 - 3} = -\frac{y}{3}$. The sum of the slopes is $\frac{y}{3} + \frac{y}{3} = 0$.

The correct choice is B.

6. Since $PQRS$ is a rectangle, its angles measure 90°. The triangles that include the marked angles are right triangles. Write an equation for the measure of $\angle PSR$, using expressions for the unmarked angles on either side of the angle of x°.

$$90 = (90 - a) + x + (90 - b)$$
$$0 = 90 - a - b + x$$
$$a + b = 90 + x$$

The correct choice is A.

7. Simplify the fraction. One method is to multiply both numerator and denominator by $\frac{y^2}{y^2}$.

$$\frac{y - \frac{1}{y}}{1 - \frac{2}{y} + \frac{1}{y^2}} = \frac{y^2}{y^2}\left(\frac{y - \frac{1}{y}}{1 - \frac{2}{y} + \frac{1}{y^2}}\right)$$
$$= \frac{y^3 - y}{y^2 - 2y + 1}$$
$$= \frac{y(y^2 - 1)}{(y - 1)(y - 1)}$$
$$= \frac{y(y - 1)(y + 1)}{(y - 1)(y - 1)}$$
$$= \frac{y^2 + y}{y - 1}$$

Another method is to write both the numerator and denominator as fractions, and then simplify.

$$\frac{y - \frac{1}{y}}{1 - \frac{2}{y} + \frac{1}{y^2}} = \frac{\frac{y^2 - 1}{y}}{\frac{y^2 - 2y + 1}{y^2}}$$
$$= \frac{y^2 - 1}{y}\left(\frac{y^2}{y^2 - 2y + 1}\right)$$
$$= \frac{y(y - 1)(y + 1)}{(y - 1)(y - 1)}$$
$$= \frac{y^2 + y}{y - 1}$$

The correct choice is A.

8. Since the triangles are similar, use a proportion with corresponding sides of the two triangles.

$$\frac{BC}{AC} = \frac{BD}{AE}$$
$$\frac{2}{2 + 3} = \frac{4}{AE}$$
$$2AE = 4(2 + 3)$$
$$AE = 10$$

The correct choice is E.

9. Since the volume V varies directly with the temperature T, the volume and temperature satisfy the equation $V = kT$, where k is a constant. When $V = 12$, $T = 60$. So $12 = 60k$, or $k = \frac{1}{5}$.
The relationship is $V = \frac{1}{5}T$.
To find the volume when the temperature is 70°, substitute 70 for T in the equation $V = \frac{1}{5}T$.
$V = \frac{1}{5}(70)$ or 14. The volume of the balloon is 14 in^3.

The correct choice is C.

10. Two sides have the same length. The lengths of all sides are integers. The third side is 13. From Triangle Inequality, the sum of the lengths of any two sides must be greater than the length of the third side. Let s be the length of the other two sides. Write and solve an inequality.

$$2s > 13$$
$$s > 6.5$$

The length of the sides must be greater than 6.5. But the length of the sides must be an integer. The smallest integer greater than 6.5 is 7. The answer is 7. If you answered 6.5, you did not find an integer. If you answered 6, you found a number that is less than 6.5.